Problem 1 (20 marks)
Motion along the x axis: Determine the xp time transformation function $\langle x, t|p, t_0\rangle$ for the Hamilton operator

$$H = \frac{1}{2M(t)}P^2 \,,$$

which has a time-dependent mass $M(t)$.

Problem 2 (20 marks)
Harmonic oscillator with ladder operators A, $A^\dagger$ and Fock state kets $|n\rangle$: Operator V is defined by

$$V = \sum_{n=0}^{\infty} |n+1\rangle\langle n| \,.$$

Verify that $V^\dagger V = 1$, and show that V is <u>not</u> unitary. Determine $W(A^\dagger A)$ such that $V = A^\dagger\, W(A^\dagger A)$.

Problem 3 (20 marks)
As usual we denote by A and $A^\dagger$ the ladder operators of a harmonic oscillator. Show that the normally-ordered form of an arbitrary function $F(A^\dagger A)$ of the product $A^\dagger A$ is given by

$$F(A^\dagger A) = \sum_{n=0}^{\infty} \frac{F(n)}{n!} A^{\dagger n} \mathrm{e}^{-A^\dagger ; A} A^n \,.$$

Use this to find the normally-ordered form of $z^{A^\dagger A}$ where z is any complex number.

Problem 4 (20 marks)
What are the eigenvalues and their multiplicities of the Hamilton operator

$$H = \frac{1}{2M}\vec{P}^2 + \frac{1}{2}M\omega^2\vec{R}^2$$

for a three-dimensional isotropic harmonic oscillator?

Problem 5 (20 marks)
Orbital angular momentum with $\langle l, m|$ denoting the usual common eigenbras of $\vec{L}^2$ and L_3: The system is in the state described by the bra

$$\langle\;| = \frac{2}{3}\langle l=1, m=1| + \frac{1}{3}\langle l=1, m=0| + \frac{2}{3}\langle l=1, m=-1| \,.$$

Find the spreads of L_1, L_2, and L_3, and verify that each pair of them obeys the respective uncertainty relation.