

Write answers on this side of the paper only.

[1] The Euler Lagrange equation

$$2 \left(\frac{d}{dx} \right)^2 f(x) = \frac{15}{2} \frac{f(x)^2}{\sqrt{x}}$$

is solved by $f(x) = x^{-3/2}$ which obeys the constraints $f(1) = 1$, $f(\infty) = 0$.
The integral has the value

$$\int_1^{\infty} dx \left(\frac{9}{4} + \frac{5}{2} \right) x^{-5} = \frac{19}{16}$$

[2] In view of

$$B^2: z \mapsto -z$$

$$B^3: z \mapsto -iz$$

$$B^4: z \mapsto z; \quad B^4 = E = BB^3 = B^2B^2 = B^3B$$

$$AB: z \mapsto -iz^* \quad | \quad A^2 = E$$

$$AB^2: z \mapsto -z^*$$

$$AB^3: z \mapsto iz^*;$$

$$BA = AB^3$$

$$B^2A = BAB^3 = AB^2$$

$$ABA = A^2B^3 = B^3$$

$$BAB = AB^4 = A$$

$$BAB^2 = AB$$

$$B^3A = BAB^2 = AB$$

the completed table is

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	E	A	B	B ²	B ³	AB	AB ²	AB ³
E	E	A	B	B ²	B ³	AB	AB ²	AB ³
A	A	E	AB	AB ²	AB ³	B	B ²	B ³
B	B	AB ³	B ²	B ³	E	A	AB	AB ²
B ²	B ²	AB ²	B ³	E	B	AB ³	A	AB
B ³	B ³	AB	E	B	B ²	AB ²	AB ³	A
AB	AB	B ³	AB ²	AB ³	A	E	B	B ²
AB ²	AB ²	B ²	AB ³	A	AB	B ³	E	B
AB ³	AB ³	B	A	AB	AB ²	B ²	B ³	E

(b) Subgroups with two elements consist of

E and either A, or B², or AB, or AB²,
or AB³;

Subgroups with four elements are

{E, B, B², B³} , the cyclic group,

and {E, A, B², AB²} , the vierergruppe,

and {E, B², AB, AB³} , the vierergruppe again.

(c) All are abelian.

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3 The Laplace transform $F(s) = \int_0^{\infty} dt e^{-st} f(t)$
obeys

$$F(s) + \frac{d}{ds} (sF(s) - f(0)) = 2F(s)^2$$

$$\text{or } s \frac{d}{ds} F(s) + 2F(s) = 2F(s)^2$$

$$\text{and } \int_0^{\infty} ds F(s) = \pi.$$

The differential equation has the general
solution

$$F(s) = \frac{a}{a+s^2}$$

and we need $a > 0$ for the integral to
converge. Since

$$\int_0^{\infty} ds \frac{a}{a+s^2} = \frac{\pi}{2} \sqrt{a}$$

we find $a = 4$, so that

$$F(s) = 2 \frac{2}{s^2 + 2^2}$$

and $f(t) = 2 \sin(2t)$.

