

Write answers on this side of the paper only.

Do not write on either margin

4-wave vectors of the plane waves. In the rest-frame of the mirror we would have

$$\begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ for the 4-velocity, and } \frac{\omega_0}{c} \begin{pmatrix} 1 \\ \sin \theta_0 \\ 0 \\ \mp \cos \theta_0 \end{pmatrix}$$

for the two 4-wave vectors.

The Lorentz-transformation into the rest frame of the mirror is

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \gamma \begin{pmatrix} 1 & 0 & 0 & -v/c \\ 0 & 1/\gamma & 0 & 0 \\ 0 & 0 & 1/\gamma & 0 \\ -v/c & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix},$$

as verified by

$$\begin{pmatrix} \gamma c \\ 0 \\ 0 \\ \gamma v \end{pmatrix} \rightarrow \gamma^2 \begin{pmatrix} 1 - v^2/c^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

So,

$$\begin{aligned} \frac{\omega_0}{c} \begin{pmatrix} 1 \\ \sin \theta_0 \\ 0 \\ -\cos \theta_0 \end{pmatrix} &= \gamma \begin{pmatrix} 1 & 0 & 0 & -v/c \\ 0 & 1/\gamma & 0 & 0 \\ 0 & 0 & 1/\gamma & 0 \\ -v/c & 0 & 0 & 1 \end{pmatrix} \frac{\omega}{c} \begin{pmatrix} 1 \\ \sin \theta \\ 0 \\ -\cos \theta \end{pmatrix} \\ &= \gamma \frac{\omega}{c} \begin{pmatrix} 1 + \frac{v}{c} \cos \theta \\ \frac{1}{\gamma} \sin \theta \\ 0 \\ -\cos \theta - v/c \end{pmatrix} \end{aligned}$$

and, likewise,

$$\frac{\omega_0}{c} \begin{pmatrix} 1 \\ \sin \theta_0 \\ 0 \\ \cos \theta_0 \end{pmatrix} = \gamma \frac{\omega'}{c} \begin{pmatrix} 1 - \frac{v}{c} \cos \theta' \\ \frac{1}{\gamma} \sin \theta' \\ 0 \\ \cos \theta' - v/c \end{pmatrix},$$

and upon equating the time components and

