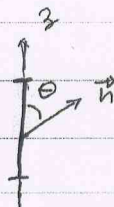


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1] The expression for  $\frac{dP}{d\Omega}(t)$  on page 76 of the notes applies, whereby here



$$\int (d\vec{r}') \frac{\partial}{\partial t} \vec{j}(\vec{r}', t_r)$$

$$= I \vec{e}_z \int_{-\pi c/\omega}^{\pi c/\omega} dz' \sin(\omega z'/c) \frac{\partial}{\partial t} \cos(\omega(t - \frac{r}{c} + \frac{z'}{c} \cos\theta))$$

$$= -c I \vec{e}_z \int_{-\pi}^{\pi} d\phi \sin\phi \sin(\omega t_e + \phi \cos\theta)$$

with  $t_e = t - \frac{r}{c}$  (emission time), and  $\phi = \frac{\omega}{c} z'$ .

Of

$$\sin(\omega t_e + \phi \cos\theta) = \sin(\omega t_e) \cos(\phi \cos\theta) + \cos(\omega t_e) \sin(\phi \cos\theta)$$

only the 2nd term contributes to the  $\phi$  integral, giving

$$\int (d\vec{r}') \frac{\partial}{\partial t} \vec{j}(\vec{r}', t_r)$$

$$= \frac{1}{2} c I \vec{e}_z \cos(\omega t_e) \int_{-\pi}^{\pi} d\phi [\cos(\phi + \phi \cos\theta) - \cos(\phi - \phi \cos\theta)]$$

$$= c I \vec{e}_z \cos(\omega t_e) \left[ \frac{\sin(\pi + \pi \cos\theta)}{1 + \cos\theta} - \frac{\sin(\pi - \pi \cos\theta)}{1 - \cos\theta} \right]$$

$$= -2c I \vec{e}_z \cos(\omega t_e) \frac{\sin(\pi \cos\theta)}{1 - (\cos\theta)^2}$$

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and with  $|\vec{n} \times \vec{e}_z|^2 = (\sin \theta)^2$ , we obtain

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3} (2cI)^2 \omega(\omega t_e)^2 \left( \frac{\sin(\pi \omega \theta)}{\sin \theta} \right)^2.$$

Upon averaging over one or more periods of the oscillation,  $\omega(\omega t_e)^2 \rightarrow \frac{1}{2}$ , this yields the result

$$\frac{dP}{d\Omega} = \frac{I^2}{2\pi c} \left( \frac{\sin(\pi \cos \theta)}{\sin \theta} \right)^2.$$

For  $\theta = 0, \frac{\pi}{2}, \pi$ , the argument of  $\sin(\pi \cos \theta)$  is  $\pi, 0$ , and  $-\pi$ , respectively, so that the numerator vanishes. For  $\theta = 0, \pi$  we get an expression of the form  $\frac{0}{0}$  which requires attention, but since the right-hand side does not change under the replacement  $\theta \rightarrow \pi - \theta$ , it is enough to consider the limit  $\theta \rightarrow 0$ . Applying l'Hopital's rule,

$$\lim_{\theta \rightarrow 0} \frac{\sin(\pi \cos \theta)}{\sin \theta} = \lim_{\theta \rightarrow 0} \frac{-\omega(\pi \cos \theta) \pi \sin \theta}{\cos \theta} = 0,$$

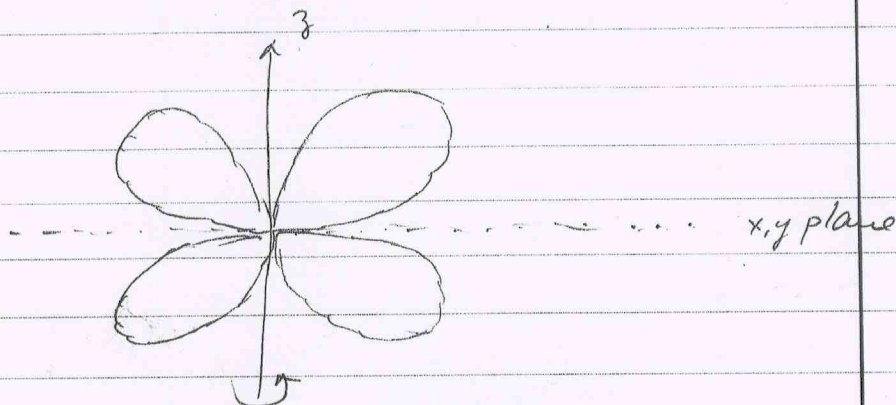
establishes  $\frac{dP}{d\Omega} = 0$  for  $\theta = 0, \frac{\pi}{2}, \pi$  and

$\frac{dP}{d\Omega} \neq 0$  for all other  $\theta$  values. We thus

have the following rough picture for the

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angular distribution:



which is rotationally symmetric around the z axis along which the antenna is stretched out.

[2] The expansion for  $\frac{dE(\omega)}{d\Omega}$  on page 104 applies, whereby here

$$\vec{j}(\vec{r}, t) = \begin{cases} 0 & \text{for } t < 0, \\ -e\vec{v} \delta(\vec{r} - \vec{v}t) & \text{for } t > 0, \end{cases}$$

so that

$$\begin{aligned} \vec{j}(\vec{k}, \omega) &= \int (d\vec{r}) e^{-i\vec{k} \cdot \vec{r}} \int_0^{\infty} dt e^{i\omega t} [-e\vec{v} \delta(\vec{r} - \vec{v}t)] \\ &= -e\vec{v} \int_0^{\infty} dt e^{i\omega (1 - \frac{v}{c} \cos \theta) t} \end{aligned}$$

for  $\vec{k} \cdot \vec{v} t = \frac{\omega}{c} vt \cos \theta$ .

