

Write answers on this side of the paper only.

□ We use the infinitesimal version of the 4-dyadic of (3.1.10),

$$L(\delta\vec{v}) = \begin{pmatrix} 1 & \delta\vec{v}^T/c \\ \delta\vec{v}/c & \mathbf{I} \end{pmatrix},$$

and first get

$$L(\delta\vec{v}_2) L(\delta\vec{v}_1) = \begin{pmatrix} 1 + \delta\vec{v}_2 \cdot \delta\vec{v}_1 / c^2 & (\delta\vec{v}_2 + \delta\vec{v}_1)^T / c \\ (\delta\vec{v}_2 + \delta\vec{v}_1) / c & \mathbf{I} + \delta\vec{v}_2 \delta\vec{v}_1^T / c^2 \end{pmatrix}$$

as well as

$$L(-\delta\vec{v}_2) L(-\delta\vec{v}_1) = \begin{pmatrix} 1 + \delta\vec{v}_2 \cdot \delta\vec{v}_1 / c^2 & -(\delta\vec{v}_2 + \delta\vec{v}_1)^T / c \\ -(\delta\vec{v}_2 + \delta\vec{v}_1) / c & \mathbf{I} + \delta\vec{v}_2 \delta\vec{v}_1^T / c^2 \end{pmatrix}$$

and thus establish

$$\begin{aligned} & L(-\delta\vec{v}_2) L(-\delta\vec{v}_1) L(\delta\vec{v}_2) L(\delta\vec{v}_1) \\ &= \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & \mathbf{I} + (\delta\vec{v}_2 \delta\vec{v}_1^T - \delta\vec{v}_1 \delta\vec{v}_2^T) / c^2 \end{pmatrix}, \end{aligned}$$

so that the effect on t and \vec{r} is

$$\begin{pmatrix} ct \\ \vec{r} \end{pmatrix} \rightarrow \begin{pmatrix} ct \\ \vec{r} + (\delta\vec{v}_2 \delta\vec{v}_1^T - \delta\vec{v}_1 \delta\vec{v}_2^T) \cdot \vec{r} / c^2 \end{pmatrix}$$

or $\delta t = 0$ and

$$\delta\vec{r} = \frac{1}{c^2} (\delta\vec{v}_1 \times \delta\vec{v}_2) \times \vec{r} = \vec{\delta\phi} \times \vec{r},$$

which is a rotation by the infinitesimal vectorial
angle $\vec{\delta\phi} = \frac{1}{c^2} (\delta\vec{v}_1 \times \delta\vec{v}_2)$.

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[2] We have

$$\begin{aligned} \partial_\nu T^{\mu\nu} &= \partial_\nu \left(\frac{1}{4\pi} F^{\mu\lambda} F^\nu{}_\lambda - \frac{1}{16\pi} g^{\mu\nu} F^{\kappa\lambda} F_{\kappa\lambda} \right) \\ &= \frac{1}{4\pi} F^{\mu\lambda} \partial_\nu F^\nu{}_\lambda + \frac{1}{4\pi} F_{\nu\lambda} \partial^\nu F^{\mu\lambda} \\ &\quad - \frac{1}{16\pi} \partial^\mu (F^{\kappa\lambda} F_{\kappa\lambda}) \end{aligned}$$

where $\partial_\nu F^\nu{}_\lambda = -\frac{4\pi}{c} j_\lambda$ (see (4.2.13)) and, as a consequence of (4.2.16),

$$\begin{aligned} F_{\nu\lambda} \partial^\nu F^{\mu\lambda} &= \frac{1}{2} F_{\nu\lambda} \partial^\nu F^{\mu\lambda} - \frac{1}{2} F_{\nu\lambda} \partial^\lambda F^{\nu\mu} - \frac{1}{2} F_{\nu\lambda} \partial^\mu F^{\lambda\nu} \\ &= \frac{1}{2} \underbrace{(F_{\nu\lambda} \partial^\nu F^{\mu\lambda} - F_{\lambda\nu} \partial^\lambda F^{\mu\nu})}_{=0} - \frac{1}{4} \partial^\mu (F_{\nu\lambda} F^{\lambda\nu}) \\ &= \frac{1}{4} \partial^\mu (F^{\lambda\nu} F_{\lambda\nu}), \end{aligned}$$

so that

$$\begin{aligned} \partial_\nu T^{\mu\nu} &= -F^{\mu\lambda} \frac{1}{c} j_\lambda + \frac{1}{16\pi} \partial^\mu (F^{\lambda\nu} F_{\lambda\nu} - F^{\kappa\lambda} F_{\kappa\lambda}) \\ &= -F^{\mu\nu} \frac{1}{c} j_\nu, \end{aligned}$$

indeed, using ordinary matrix multiplication, we evaluate

$$\begin{aligned} F^{\mu\nu} \frac{1}{c} j_\nu &= F^\mu{}_\nu \frac{1}{c} j^\nu \\ &= \begin{pmatrix} 0 & \vec{E}^T \\ \vec{E} & -\vec{B} \times \vec{I} \end{pmatrix} \begin{pmatrix} \rho \\ \frac{1}{c} \vec{j} \end{pmatrix} = \begin{pmatrix} \frac{1}{c} \vec{j} \cdot \vec{E} \\ \rho \vec{E} + \frac{1}{c} \vec{j} \times \vec{B} \end{pmatrix} \end{aligned}$$

