Problem 1 (25 marks)
Consider the following sequence of four infinitesimal Lorentz transformations:

first by $\delta\vec{v}_1$, then by $\delta\vec{v}_2$, next by $-\delta\vec{v}_1$, finally by $-\delta\vec{v}_2$.

Keeping terms that are at most first order in $\delta\vec{v}_1$ or $\delta\vec{v}_2$ or both, show that the total transformation amounts to

$$t \to t\,,\ \vec{r} \to \vec{r} + \vec{\delta\phi} \times \vec{r} \quad \text{or} \quad \delta t = 0\,,\ \delta\vec{r} = \vec{\delta\phi} \times \vec{r}$$

with $\vec{\delta\phi} = \dfrac{1}{c^2}(\delta\vec{v}_1 \times \delta\vec{v}_2)$. What does the total transformation mean in geometrical terms?

Problem 2 (25 marks)
Exploit the definitions of $F^{\mu\nu}$ and $T^{\mu\nu}$ in (4.2.2) and (4.2.25), respectively, to verify the statement of (4.2.31) on page 52 of the notes, that is:

$$\partial_\nu T^{\mu\nu} = -F^{\mu\nu}\frac{1}{c}j_\nu\,.$$

Relate the right-hand side to the 4-force density on page 29.

Problem 3 (25 marks)
A point charge e is moving on a circle of radius R with constant speed v, so that

$$x(t) = R\cos(vt/R)\,,\ y(t) = R\sin(vt/R)\,,\ z(t) = 0$$

are the charge's cartesian coordinates as a function of time t. Find the retarded potentials for points on the z axis. Which components of $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ can you infer from this limited knowledge of the potentials?

Problem 4 (25 marks)
A bit more realistic than the antenna model of Section 6.6 in the notes is the model defined by the electric current density

$$\vec{j}(\vec{r},t) = \vec{e}_z I\delta(x)\delta(y)\eta(L^2 - 4z^2)\cos(\pi z/L)\cos(\omega t)\,.$$

Find the corresponding charge density $\rho(\vec{r},t)$. Then calculate the electric dipole moment $\vec{d}(t)$, the magnetic dipole moment $\vec{\mu}(t)$, and the electric quadrupole moment $\overleftrightarrow{Q}(t)$. Use them to determine the time-averaged total radiated power $\displaystyle\int \mathrm{d}\Omega\,\frac{\mathrm{d}P}{\mathrm{d}\Omega}$ in accordance with the Larmor formula (6.3.16).