

Write answers on this side of the paper only.

$$1) P(T) = \int d\Omega \frac{dPCT}{d\Omega}$$

$$\stackrel{(9.8.4)}{=} \frac{\omega_0}{16\pi} \frac{e^2}{R} \frac{v}{c} 2\pi \int_0^\pi d\theta \sin\theta \left[\frac{1+3\left(\frac{v}{c}\right)^2}{\left(1-\frac{v}{c}\cos\theta\right)^3} - \frac{1-\left(\frac{v}{c}\right)^2}{\left(1-\frac{v}{c}\cos\theta\right)^4} \right]$$

$$= \frac{\omega_0}{8} \frac{e^2}{R} \frac{v}{c} \left[-\frac{1}{2\frac{v}{c}} \frac{1+3\left(\frac{v}{c}\right)^2}{\left(1-\frac{v}{c}\cos\theta\right)^2} + \frac{1}{3\frac{v}{c}} \frac{1-\left(\frac{v}{c}\right)^2}{\left(1-\frac{v}{c}\cos\theta\right)^3} \right]_{\theta=0}^{\theta=\pi}$$

$$= \frac{\omega_0}{8} \frac{e^2}{R} \frac{v}{c} \frac{1}{\left(1-\left(\frac{v}{c}\right)^2\right)^2} \left[2\left(1+3\left(\frac{v}{c}\right)^2\right) - \left(2+\frac{2}{3}\left(\frac{v}{c}\right)^2\right) \right]$$

$$= \frac{2}{3} \omega_0 \frac{e^2}{R} \left(\frac{v}{c}\right)^3 \frac{1}{\left(1-\left(\frac{v}{c}\right)^2\right)^2}$$

$$= \frac{2}{3} \omega_0 \frac{e^2}{R} \left(\frac{v}{c}\right)^3 \left(\frac{E}{mc^2}\right)^4, \text{ indeed.}$$

2) We have $\vec{d}(t) = \frac{e-1}{e+2} R^3 \vec{E}(t)$ and $\frac{d^2 \vec{E}(t)}{dt^2} = -\omega^2 \vec{E}(t)$,
so that Larmor's formula (6.3.6) gives

$$P = \frac{2}{3c^3} \omega^4 \left(\frac{e-1}{e+2} R^3\right)^2 \underbrace{|\vec{E}(t)|^2}_{= \frac{4\pi}{c} |\vec{S}|} = \frac{8\pi}{3} |\vec{S}|$$

with

$$\vec{S} = \frac{8\pi}{3} \left(\frac{\omega}{c}\right)^4 \left(\frac{e-1}{e+2} R^3\right)^2$$

Write answers on this side of the paper only.

[3] (a) We apply (11.2.2),

$$E(x, z) \approx -\frac{E_0}{2\pi} \frac{\partial}{\partial z} \int_0^{\infty} dx' \int_{-\infty}^{\infty} dy' \frac{e^{ik\sqrt{(x-x')^2 + y'^2 + z^2}}}{\sqrt{(x-x')^2 + y'^2 + z^2}}$$

and recognize that the y' integral is the same as in (11.6.23), so that

$$E(x, z) \approx -\frac{E_0}{2\pi} \frac{\partial}{\partial z} \int_0^{\infty} dx' \sqrt{\frac{2\pi i}{k\sqrt{(x-x')^2 + z^2}}} e^{ik\sqrt{(x-x')^2 + z^2}}$$

$$\approx -\frac{E_0}{2\pi} \sqrt{\frac{2\pi i}{kz}} ik e^{ikz} \int_0^{\infty} dx' e^{i\frac{1}{2}k\frac{(x-x')^2}{z}}$$

after employing the usual approximations. substitute $x' = x + \sqrt{\pi z/k} t$ to arrive at

$$\frac{E(x, z)}{E_0} \approx \frac{e^{ikz}}{\sqrt{2i}} \int_{-T}^{\infty} dt e^{i\frac{\pi}{2}t^2} = \frac{e^{ikz}}{\sqrt{2i}} \int_{-\infty}^T dt e^{i\frac{\pi}{2}t^2}$$

with $T = \sqrt{\frac{k}{\pi z}} x = \sqrt{\frac{2}{\lambda z}} x$, so that

$$\left| \frac{E(x, z)}{E_0} \right|^2 = \frac{1}{2} \left| \int_{-\infty}^T dt e^{i\frac{\pi}{2}t^2} \right|^2$$

$$= \frac{1}{2} \left| \frac{1+i}{2} + \underbrace{\int_0^T dt e^{i\frac{\pi}{2}t^2}}_{F(T)} \right|^2$$

or

$$\left| \frac{E(x, z)}{E_0} \right|^2 = \frac{1}{4} \left| 1 + (1-i) F(T) \right|^2$$

