

Write answers on this side of the paper only.

(a) According to (3.5.7) we have here  $\vec{A}(\vec{r}, t) = \frac{\vec{v}}{c} \Phi(\vec{r}, t)$ , so that

$$\begin{aligned}\vec{B} &= \nabla \times \vec{A} = \frac{\vec{v}}{c} \times (-\nabla \Phi) = \frac{\vec{v}}{c} \times \left( -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} - \nabla \Phi \right) \\ &= \frac{\vec{v}}{c} \times \vec{E}.\end{aligned}$$

$\hookrightarrow \propto \vec{v}/c$

(b) Lorentz force  $\vec{F} = e \vec{E}(\vec{v}t + \vec{a}, t) + e \frac{\vec{v}}{c} \times \vec{B}(\vec{v}t + \vec{a}, t)$

$$= e \left[ \vec{E} + \frac{\vec{v}}{c} \times \left( \frac{\vec{v}}{c} \times \vec{E} \right) \right]$$

$$= e \left( 1 - \frac{v^2}{c^2} \right) \vec{E} + e \frac{\vec{v}}{c} \frac{\vec{v}}{c} \cdot \vec{E},$$

and from Exercise 13 we recall that

$$\vec{E}(\vec{r}, t) = \frac{\gamma e}{R^3} (\vec{r} - \vec{v}t)$$

with  $R = \sqrt{x^2 + y^2 + \gamma^2 (z - vt)^2}$ , so that

$$\vec{E}(\vec{v}t + \vec{a}, t) = \frac{\gamma e}{R^3} \vec{a} \quad \text{with } R = \sqrt{\vec{a}_\perp^2 + \gamma^2 \vec{a}_\parallel^2}.$$

Together, these say that  $\left( \frac{\vec{v}}{c} \frac{\vec{v}}{c} \cdot \vec{a} = \left( \frac{v}{c} \right)^2 \vec{a}_\parallel = \frac{\gamma^2 - 1}{\gamma^2} \vec{a}_\parallel \right)$

$$\vec{F} = \frac{e^2}{\gamma R^3} \vec{a}_\perp + \frac{\gamma e^2}{R^3} \vec{a}_\parallel.$$

(c) If  $\vec{a} = \vec{a}_\perp$ ,  $\vec{a}_\parallel = 0$ , then  $R = |\vec{a}_\perp|$  and  $\vec{F} = \frac{e^2 \vec{a}}{\gamma a^3}$ .

There is no Lorentz contraction perpendicular to  $\vec{v}$ , so that  $a$  is also the distance in the rest frame. It follows that the force is smaller by a factor  $\frac{1}{\gamma}$  than the force in the rest frame.

By contrast, if  $\vec{a} = \vec{a}_\parallel$ ,  $\vec{a}_\perp = 0$ , then  $R = \gamma |\vec{a}_\parallel| = \gamma a$

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and  $\vec{F} = e^2 \frac{(\gamma \vec{a})}{(\gamma a)^3}$  where  $\gamma \vec{a}$  is the distance vector in the rest frame and  $\gamma a$  is its length, so that the force is the same as in the rest frame.

[2] Since the charged particle moves parallel to the surface of the dielectric, total internal reflection prevents the Čerenkov radiation from leaving the dielectric. Answer: The fraction is zero.

[3] We apply the Huygens approximation and get, as a simple modification of (11.3.3)

$$E_x(\vec{r}) \cong -E_0 \frac{e^{ikr}}{r} \frac{i}{k\theta^2} \int_{ka\theta}^{kb\theta} dt t J_0(t)$$

$$= -E_0 \frac{e^{ikr}}{r} \frac{i}{k\theta^2} [kb\theta J_1(kb\theta) - ka\theta J_1(ka\theta)]$$

and, see (11.3.8),

$$\frac{d\sigma}{d\Omega} = r^2 \left| \frac{E_x(\vec{r})}{E_0} \right|^2 \cong \left| \frac{b}{\theta} J_1(kb\theta) - \frac{a}{\theta} J_1(ka\theta) \right|^2,$$

valid for  $\theta \ll 1$ , which are the relevant  $\theta$  values.

[4] Equation of motion is  $\frac{d}{dt} \vec{p} = e \frac{\vec{v}}{c} \times \vec{B}$  with  $\vec{p} = m\gamma \vec{v}$ ,

$$\text{or } \frac{d}{dt} \vec{v} = 0 \text{ for } z < 0 \text{ and } z > L$$

and

$$\frac{d}{dt} \vec{v} = \frac{eB}{\gamma mc} \begin{pmatrix} -v_z \sin(k_0 z) \\ v_z \cos(k_0 z) \\ v_x \sin(k_0 z) - v_y \cos(k_0 z) \end{pmatrix} \text{ for } 0 < z < L.$$

