

Question... 1/5...

Write answers on this side of the paper only.

Do not write on either margin

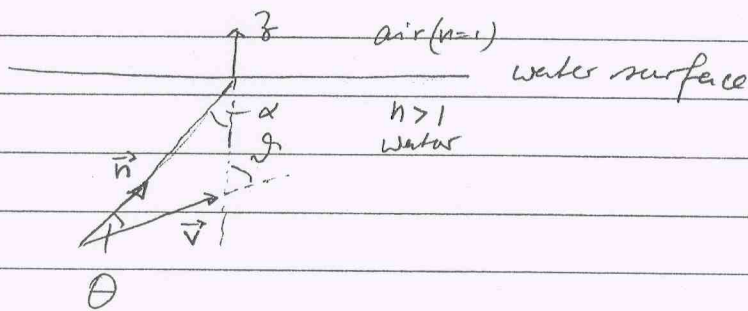
- 1 Rayleigh scattering differs from Thomson scattering only by the frequency dependent factor, so that

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ray}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Th}} \left| \frac{\omega^2}{\omega_0^2 - \omega^2 - i\gamma\omega} \right|^2$$

$$= r_0^2 \frac{1}{2} (1 + \cos^2\theta) \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}$$

2

(a)



The angle of incident α must be less than the critical angle for total internal reflection,

$$\sin \alpha < \frac{1}{n} = \sin \alpha_0$$

The angle ϕ between \vec{v} of the electron and the normal surface vector \hat{e}_z is denoted by ϕ , so that Cherenkov radiation can get across the surface if

$$\phi = \theta + \alpha < \theta + \alpha_0$$

where $\cos \theta = \frac{c}{nv}$ (Cherenkov) and $\sin \alpha_0 = \frac{1}{n}$ (Snell).

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(b) We would need $\theta + \alpha_0 > \frac{\pi}{2}$, but in fact we have

$$\begin{aligned}\cos(\theta + \alpha_0) &= \cos\theta \cos\alpha_0 - \sin\theta \sin\alpha_0 \\ &= \frac{c}{nv} \sqrt{1 - \frac{1}{n^2}} - \frac{1}{n} \sqrt{1 - \left(\frac{c}{nv}\right)^2} \\ &= \frac{1}{n^2 v} \left(\sqrt{(nc)^2 - c^2} - \sqrt{(nv)^2 - c^2} \right) > 0\end{aligned}$$

which tells us that $\theta + \alpha_0 < \frac{\pi}{2}$. Answer: No.

[3] (a) The retarded time (= emission time corrected for mutual retardation) for the l th ring antenna ($l = 0, \pm 1, \pm 2, \dots, \pm M$) is

$$\begin{aligned}t_r &= t - \frac{r}{c} + \underbrace{\frac{1}{c} \vec{n} \cdot \vec{r}'}_{\text{central antenna with } l=0} - \underbrace{\frac{1}{c} \vec{n} \cdot l D \vec{e}_z}_{\text{displacement of the } l\text{th antenna}} \\ &= \left(t - \frac{r}{c} - l \frac{D}{c} \cos\theta \right) + \frac{1}{c} \vec{n} \cdot \vec{r}'\end{aligned}$$

Effectively we have the replacement

$$t_e = t - \frac{r}{c} \rightarrow t_e - l \frac{D}{c} \cos\theta$$

The integrated time derivation of the current has a time-dependence given by

$$\cos(\omega t_e + \alpha)$$

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for the central antenna ($l=0$) whereby α is some irrelevant phase that will not survive the averaging over one period. Accordingly,

$$\left(\frac{dP}{d\Omega}\right)_N = \left(\frac{dP}{d\Omega}\right)_1 \cdot 2 \left(\sum_{l=-M}^M \cos(\omega t_0 + \alpha - l \frac{\omega D}{c} \cos \theta) \right)^2$$

↑
all N
ribs

↑
central
rib only

where the sum is a geometric summation,

$$\sum_{l=-M}^M \cos(\omega t_0 + \alpha - l \frac{\omega D}{c} \cos \theta)$$

$$= \operatorname{Re} e^{i(\omega t_0 + \alpha)} \sum_{l=-M}^M e^{-il \frac{\omega D}{c} \cos \theta}$$

$$= \cos(\omega t_0 + \alpha) \frac{\sin((2M+1) \frac{\omega D}{2c} \cos \theta)}{\sin(\frac{\omega D}{2c} \cos \theta)}$$

With $N=2M+1$ and $\frac{\omega D}{2c} = \pi \frac{D}{\lambda}$, then,

$$\left(\frac{dP}{d\Omega}\right)_N = \left(\frac{dP}{d\Omega}\right)_1 \left(\frac{\sin(\pi N \frac{D}{\lambda} \cos \theta)}{\sin(\pi \frac{D}{\lambda} \cos \theta)} \right)^2$$

$$= \underbrace{\frac{\pi}{2} \frac{I^2}{c} \left(\frac{aw}{c}\right)^2 J_1\left(\frac{aw}{c} \sin \theta\right)^2}_{\text{one rib antenna}} \underbrace{\left(\frac{\sin(\pi N \frac{D}{\lambda} \cos \theta)}{\sin(\pi \frac{D}{\lambda} \cos \theta)} \right)^2}_{\text{interference of all N antennas}}$$

one rib antenna

interference of
all N antennas

