

Write answers on this side of the paper only.

□ Rotate by 90° about x-axis: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -z \\ y \end{pmatrix};$

rotate by 180° about y-axis: $\begin{pmatrix} x \\ -z \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -z \\ -y \end{pmatrix};$

rotate by 90° about z-axis: $\begin{pmatrix} -x \\ -z \\ -y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ z \\ -y \end{pmatrix};$

overall rotation is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} z \\ -x \\ -y \end{pmatrix},$$

so that all vectors with components $\begin{pmatrix} a \\ -a \\ a \end{pmatrix}$ are unchanged by the rotation. It follows that we can regard the unit vector

$$\vec{e} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

as specifying the axis of rotation; equivalently, this role can be played by $-\vec{e}$.

We take a vector perpendicular to \vec{e} , such as the one with components $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ to find the rotation angle. First, the net effect is

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix},$$

then the cosine of the rotation angle is found from $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} = 2 \cos \phi,$

which gives $\cos \phi = -\frac{1}{2}$, so that $\phi = 120^\circ$ or -120° . Finally, since

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$$\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = -3 < 0,$$

the rotation around axis \vec{e} is by -120° ; or, equivalently, the rotation around axis $-\vec{e}$ is by 120° .

- [2] We use the retarded Green's function for the harmonic oscillator without damping ($\gamma=0, \omega=\omega_0$), and so have

$$x(t) = \int_{-\infty}^t dt' \frac{1}{\omega_0} \sin(\omega_0(t-t')) \frac{F(t')}{m},$$

which clearly has $x(t)=0$ for all times before the period during which $F(t) \neq 0$.

The velocity is

$$\dot{x}(t) = \int_{-\infty}^t dt' \cos(\omega_0(t-t')) \frac{F(t')}{m},$$

and the time-dependence of the force must then be such that

$$\int_{-\infty}^t dt' \sin(\omega_0(t-t')) F(t') = 0$$

and

$$\int_{-\infty}^t dt' \cos(\omega_0(t-t')) F(t') = 0$$

for all times t after the period during which $F(t) \neq 0$. Taken together these equations tell us that

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$$\int_{-\infty}^t dt' e^{i\omega_0(t-t')} F(t') = 0$$

for all late times, but then $F=0$ for even later times, so that we arrive at the compact statement

$$\int_{-\infty}^{\infty} dt e^{-i\omega_0 t} F(t) = 0.$$

That is: The Fourier component of $F(t)$ at the circular frequency of the oscillator must vanish, but other than that there are no further restrictions on $F(t)$.

□ The components of the force are

$$F_x = -\frac{\partial}{\partial x} V(x,y) = k_1 V_0 \sin(k_1 x) \cos(k_2 y),$$

$$F_y = -\frac{\partial}{\partial y} V(x,y) = k_2 V_0 \cos(k_1 x) \sin(k_2 y).$$

Therefore we have two kinds of points where $F_x=0$ and $F_y=0$, namely

(i) $\sin(k_1 x) = 0$ and $\sin(k_2 y) = 0$,

(ii) $\cos(k_1 x) = 0$ and $\cos(k_2 y) = 0$.

