Problem 1 (20 marks)
Consider the following sequence of three rotations: First around the x axis by $90°$, then around the y axis by $180°$, finally around the z axis by $90°$. The net effect is a rotation around which axis by which angle?

Problem 2 (30 marks)
A harmonic oscillator (mass m, circular frequency ω_0) is initially at rest, so that $x(t) = 0$ for $t < 0$. Then a time-dependent force $F(t)$ is applied for a finite duration T, that is: $F(t) = 0$ for $t < 0$ and $t > T$. It so happens that the oscillator is again at rest after the force has ceased to act. What does this tell you about $F(t)$?

Problem 3 (30 marks)
A point mass m is moving in the xy-plane under the influence of the force associated with the potential energy

$$V(x, y) = V_0 \cos(k_1 x) \cos(k_2 y) \, ,$$

where V_0, k_1, and k_2 are positive constants. Determine the positions at which the force vanishes and examine whether the potential energy has a maximum, a minimum, or a saddle point there. Then find the periods of the natural small-amplitude oscillations at the minima.

Problem 4 (20 marks)
Consider a vector field $\vec{A}$ that is given in terms of cylindrical coordinates, that is: $\vec{A}(\vec{r}) = A_s(s, \varphi, z)\vec{e}_s + A_\varphi(s, \varphi, z)\vec{e}_\varphi + A_z(s, \varphi, z)\vec{e}_z$. Express $\vec{\nabla} \cdot \vec{A}$ in terms of the component functions A_s, A_φ, and A_z. Then verify that your expression gives the right answer for $\vec{A}(\vec{r}) = x\vec{e}_x$ and $\vec{A}(\vec{r}) = \vec{r}$.