**Problem 1** (20=10+10 marks)
The mirror image of $\boldsymbol{r}$ with respect to the plane through $\boldsymbol{r}=0$ that is perpendicular to unit vector $\boldsymbol{e}$ is $\boldsymbol{r}-2\boldsymbol{e}\,\boldsymbol{e}\cdot\boldsymbol{r}$. If we take two successive mirror images, first for $\boldsymbol{e}_1$ and then for $\boldsymbol{e}_2$, the overall effect is a rotation.

**(a)** Show that the axis of rotation is specified by $\boldsymbol{e}_1\times\boldsymbol{e}_2$.

**(b)** Express the angle of rotation $\phi$ in terms of the angle $\alpha$ between $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, that is $\cos\alpha=\boldsymbol{e}_1\cdot\boldsymbol{e}_2$.

**Problem 2** (20 marks)
A constant force $F=mg$ is applied to a undamped harmonic oscillator (mass $m$, circular frequency $\omega_0$, damping constant $\gamma=0$) for a finite duration $T$. It so happens that the oscillator is at $x=0$ at time $t=0$ and has vanishing velocity at time $t=T$. What are the position $x(t)$ and the velocity $\dot{x}(t)$ for $0<t<T$?

**Problem 3** (20=5+15 marks)
A point mass $m$ is moving along the $x$-axis under the influence of the force associated with the potential energy

$$V(x)=-\frac{E_0}{\big[\cosh(kx)\big]^2}\,,$$

where $E_0$ and $k$ are positive constants.

**(a)** For which ranges of energy is the motion of the point mass bounded by two turning points, by one turning point, or not bounded at all?

**(b)** For energy $E$ such that there is periodic motion between two turning points, find the period $T(E)$.

Hint: The identity $(\cosh\vartheta_1)^2-(\cosh\vartheta_2)^2=(\sinh\vartheta_1)^2-(\sinh\vartheta_2)^2$ could be useful.

**Problem 4** (20=12+8 marks)
A force field $\boldsymbol{F}(\boldsymbol{r})$ has the form

$$\boldsymbol{F}(\boldsymbol{r})=f_1(r)\boldsymbol{a}+f_2(r)\boldsymbol{a}\cdot\boldsymbol{r}\,\boldsymbol{r}$$

with a constant vector $\boldsymbol{a}$ and non-singular functions $f_1(r)$ and $f_2(r)$ that depend only on the distance $r=|\boldsymbol{r}|$ from the origin of the coordinate system.

**(a)** How are $f_1(r)$ and $f_2(r)$ related if $\boldsymbol{F}(\boldsymbol{r})$ is a conservative force?

**(b)** What is the potential energy associated with such a conservative force?

Hint: The curl of a scalar field $b(\boldsymbol{r})$ times a vector field $\boldsymbol{B}(\boldsymbol{r})$ is given by the product rule, $\boldsymbol{\nabla}\times(b\boldsymbol{B})=\boldsymbol{\nabla}b\times\boldsymbol{B}+b\boldsymbol{\nabla}\times\boldsymbol{B}$.