PC1134 Lecture 12

Topic

Review on Vectors

Objectives

To become familiar with vector and its properties, representation, addition (subtraction), and products (scalar and vector) of two vectors.

Relevance

Many physical quantities are vectors (field, force, etc.). Such quantities can depend on other quantities (both scalar and vectors) and can also be variables of other functions.

Vectors

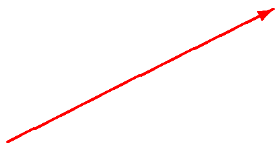
Examples of vector:

- Velocity / momentum
- Acceleration
- Force
- Fields (electric, magnetic, $\cdots$)

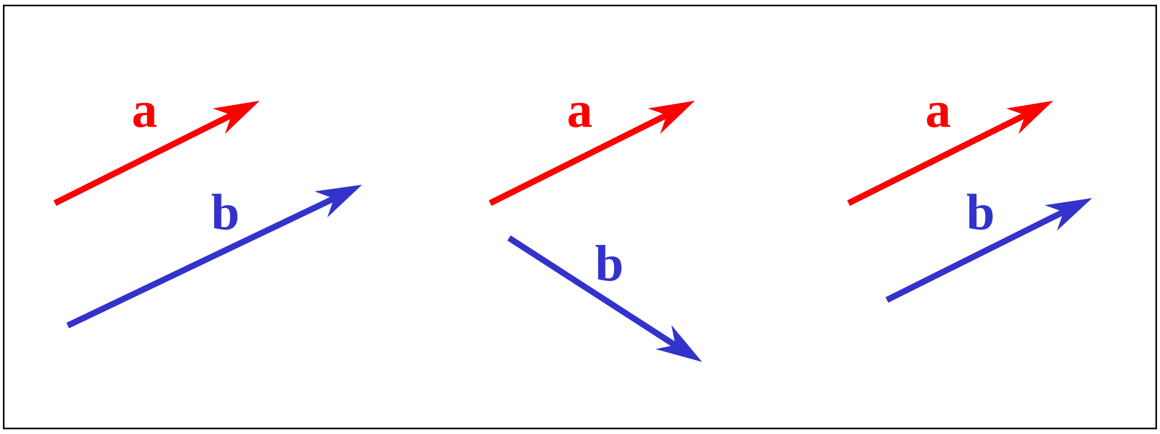
  $\cdots\cdots$

A vector consists of

- a magnitude &
- a direction



Two vectors are equal only if their magnitudes are the same and their directions are also the same.



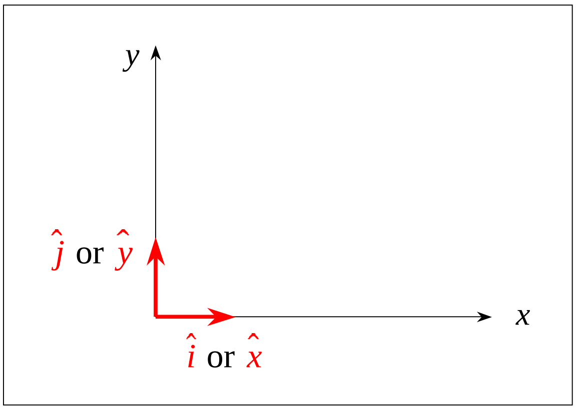
Representation of Vector

Notations:

- Bold letter (**A**) (book printing)
- $\vec{A}$ or $\tilde{A}$ or $\underline{A}$ (hand writing)

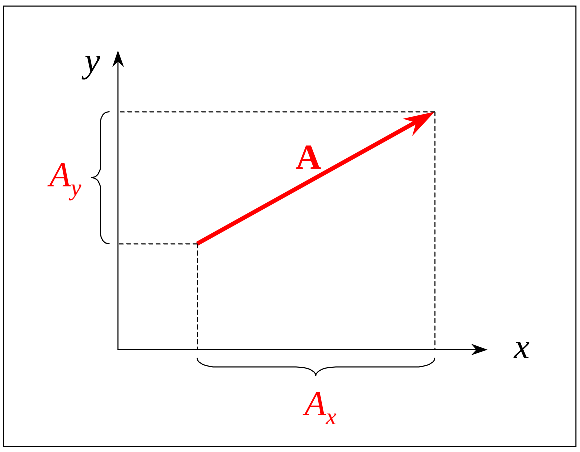
Basis vectors:

$$|\hat{x}| = 1$$
$$|\hat{y}| = 1$$
$$|\hat{z}| = 1$$



Components:

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$



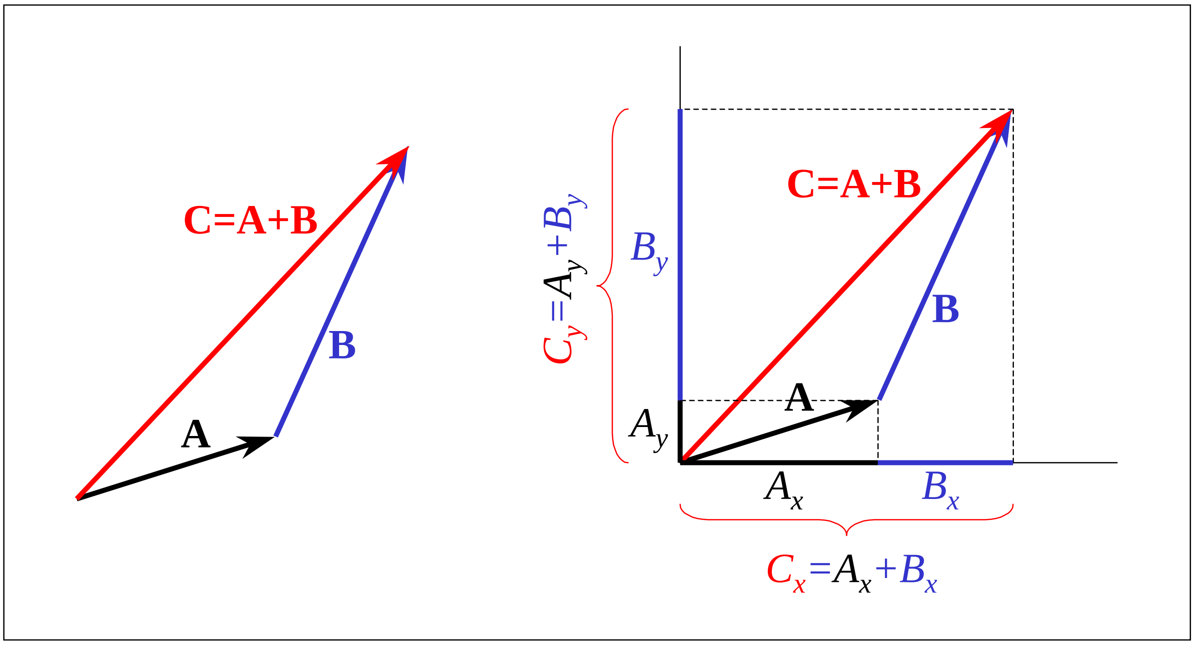
Magnitude:

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Addition of Vectors

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad \text{(commutative law)}$$

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) \qquad \text{(associative law)}$$



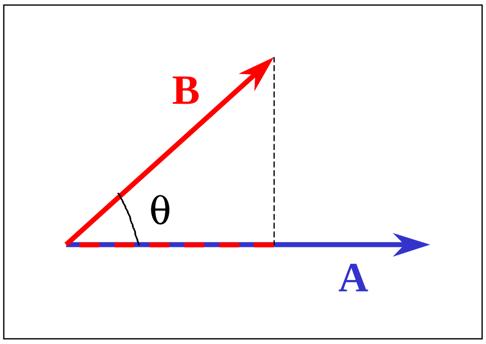
$$\begin{aligned} \vec{A} + \vec{B} &= (A_x\hat{x} + A_y\hat{y}) + (B_x\hat{x} + B_y\hat{y}) \\ &= (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} \end{aligned}$$

$$\vec{C} = C_x\hat{x} + C_y\hat{y}$$

$$\vec{C} = \vec{A} + \vec{B} \iff \begin{cases} C_x = A_x + B_x \\ C_y = A_y + B_y \end{cases}$$

Scalar Product

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$



$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$
$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$
$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad \text{(commutative law)}$$
$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} \quad \text{(distributive law)}$$

If $\vec{A}$ is perpendicular to $\vec{B}$, then

$$A_xB_x + A_yB_y + A_zB_z = 0$$

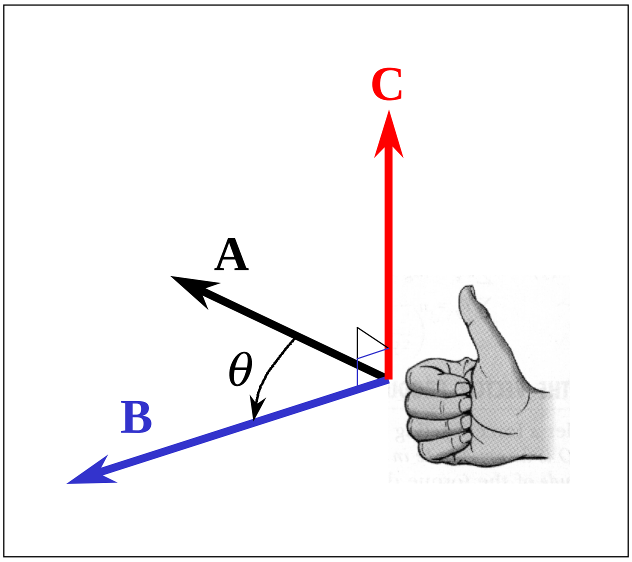
If $\vec{A}$ is parallel to $\vec{B}$, then

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

Vector Product

$$\vec{C} = \vec{A} \times \vec{B}$$

Magnitude of $\vec{C}$: $|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \theta$
Direction of $\vec{C}$: right hand rule



$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

$\vec{A} \times \vec{B} = 0$ if $\vec{A}$ and $\vec{B}$ are parallel to each other
$\vec{A} \times \vec{A} = 0$ for all $\vec{A}$

Vector Product

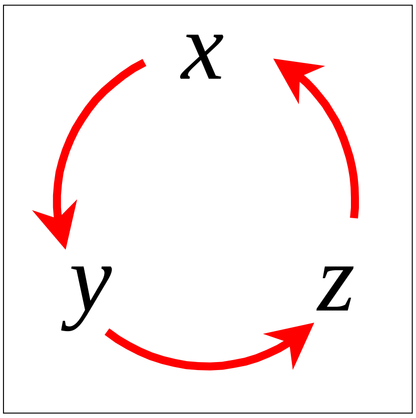
Using

$$\hat{x} \times \hat{x} = 0, \quad \hat{y} \times \hat{y} = 0, \quad \hat{z} \times \hat{z} = 0$$
$$\hat{x} \times \hat{y} = \hat{z}, \quad \hat{y} \times \hat{z} = \hat{x}, \quad \hat{z} \times \hat{x} = \hat{y}$$

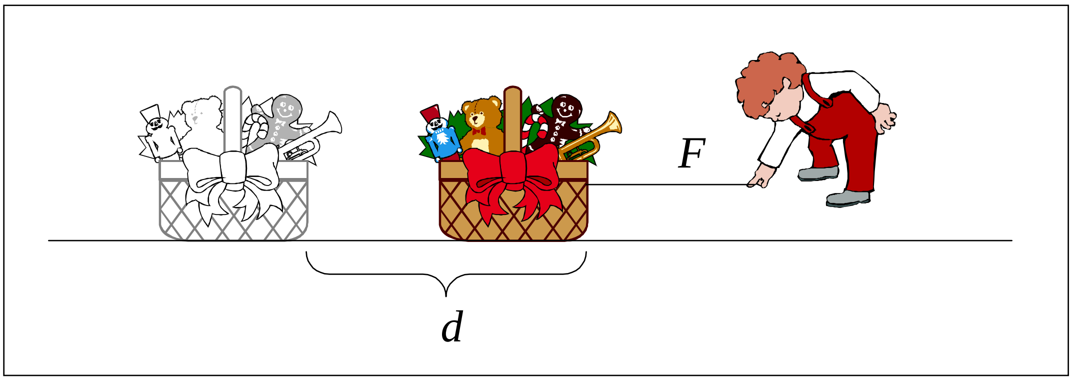
$$\begin{aligned}
\vec{A} \times \vec{B} &= (A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) \times (B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) \\
&= (A_yB_z - A_zB_y)\hat{x} + \\
&\quad (A_zB_x - A_xB_z)\hat{y} + \\
&\quad (A_xB_y - A_yB_x)\hat{z}
\end{aligned}$$

This can be written as

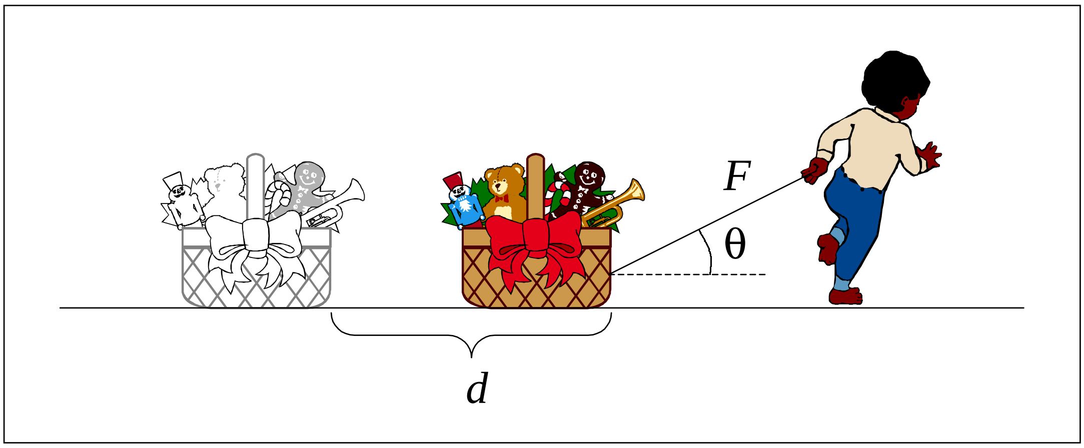
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



Work



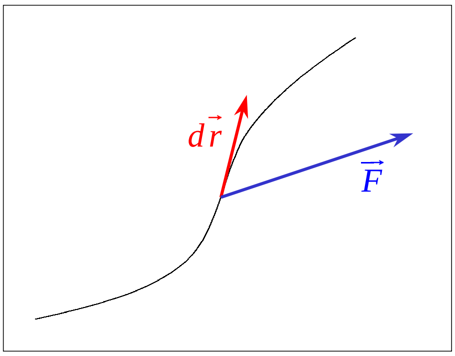
$$W = Fd$$



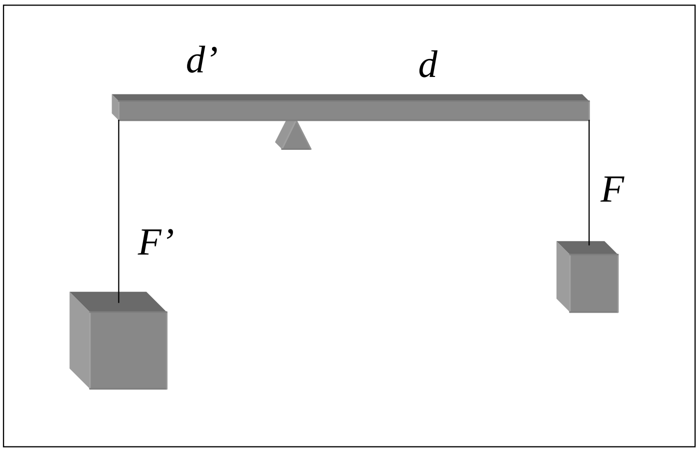
$$W = Fd\cos\theta = \vec{F}\cdot\vec{d}$$

$$dW = \vec{F}\cdot d\vec{r}$$

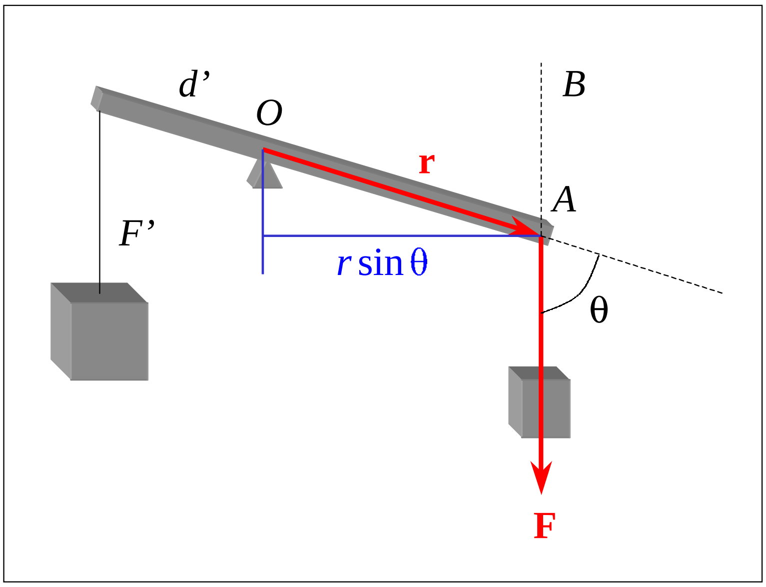
$$W = \int \vec{F}\cdot d\vec{r}$$



Torque

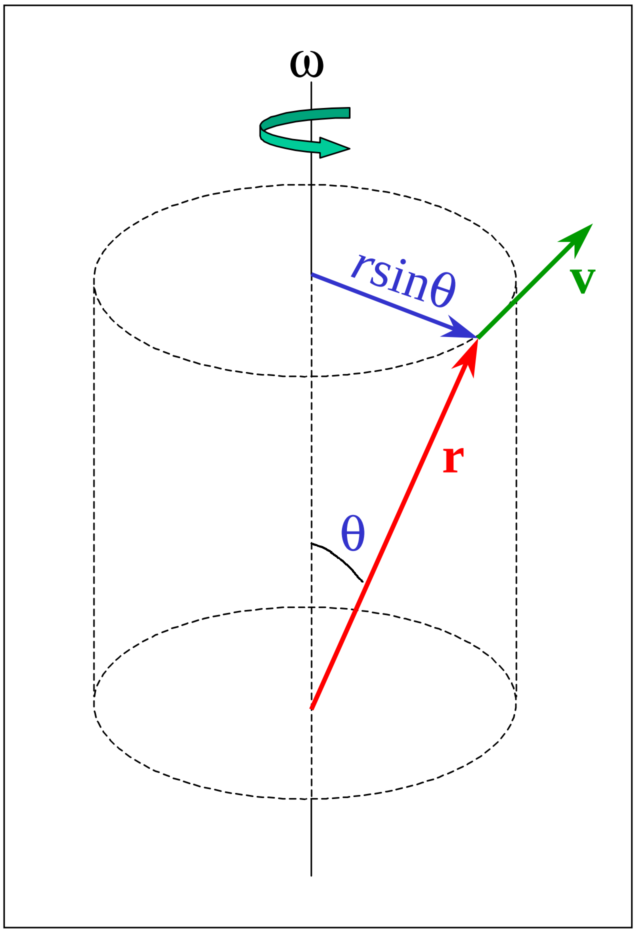


$$\tau = Fd$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

Angular Velocity



$$\vec{v} = \vec{\omega} \times \vec{r}$$