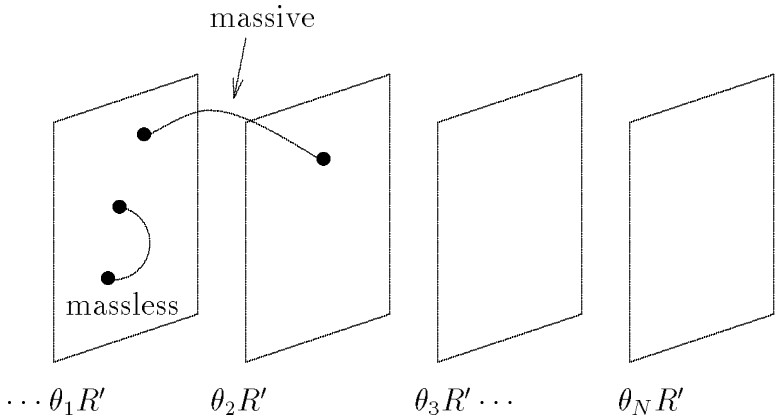
D-Brane Dynamics and Gauge Fields

We study the fluctuations of D-p branes. Recall for the case of the T-dual of a compact coordinate we have

$$\begin{aligned} M^2 &= (p^{25})^2 + \frac{1}{\alpha'}(L_0 - 1) \\ &= \{\frac{R'}{2\pi\alpha'}[2\pi n + (\theta_i - \theta_j)]\}^2 + \frac{1}{\alpha'}(L_0 - 1) \end{aligned}$$

Massless states can only arise for $n = 0$, i.e. for non-winding strings whose ends are on the same hyperplane.



Continuing in the T-dual picture, when none of the N D-branes coincide, there is just one massless $U(1)$ vector field in each D-brane with $U(1)^N$ being the unbroken group. If $m < n$ of the D-branes coincide, i.e. $\theta_1 = \theta_2 = \cdots = \theta_m$, there are m^2 massless vectors with $m^2 - m$ new massless states since open strings stretched between these branes have zero length. These m^2 massless vectors form the adjoint representation of a $U(m)$ gauge theory. This is a reflection of $U(m)$ subgroup being unbroken by the Wilson line.

It has been shown by Witten that N parallel D-branes on being coincident have low energy excitations described by a $U(N)$ susy YM-theory dimensionally reduced from $d = 10$ to $p + 1$ dimensions.

Super Yang-Mills

Let ψ be Majorana-Weyl spinor of $SO(1,9)$ acted on by 32×32 gamma matrices; let A_μ be the $U(N)$ gauge field. We then have

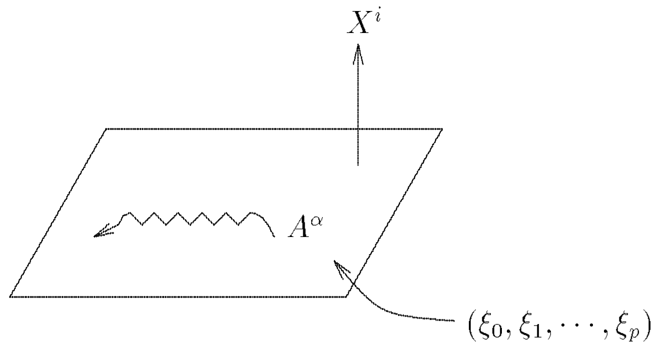
$$S = \int d^{10}\xi(-\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\mathrm{tr}\bar{\psi}\Gamma^\mu D_\mu\psi)$$

The action is invariant under

$$\begin{aligned} \delta A_\mu &= \frac{i}{2}\bar{\epsilon}\Gamma_\mu\psi \\ \delta\psi &= -\frac{1}{4}F_{\mu\nu}\Gamma^{[\mu}\Gamma^{\nu]}\epsilon \end{aligned}$$

10-D SYM has :
A_μ : 8 bosonic degrees of freedom.
ψ : 8 fermionic degrees of freedom.
16 supercharges.

To describe D-p branes low energy dynamics make all fields independent of the $9-p$ transverse (to the D-p brane) directions. The transverse oscillations of the D-p brane is described by the A^α field.



$A^\alpha(\xi)$,	$\alpha = 1, 2, \cdots p+1$:	propagate only inside the D-p brane
$A^i \equiv X^i(\xi)$,	$i = p+2, \cdots 10$:	transverse position of the D-p brane

Ignoring fermions we have

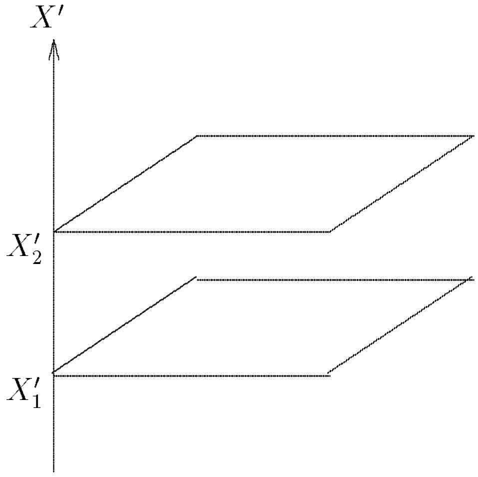
$$S = \frac{1}{4g^2} \int d^{p+1}\xi \{ -F_{\alpha\beta}F^{\alpha\beta} - (D_\alpha(A)X^i)^2 + [X^i, X^j]^2 \}$$

Classical Vacua

- Fermions vanish.
- X^i's are constant and
- $[X^i, X^j] = 0$

Thus we can simultaneously diagonalize all the X^i's.

$$X^i = \begin{pmatrix} X_1^i & & & & \\ & X_2^i & & & \\ & & . & & \\ & & & . & \\ & & & & X_N^i \end{pmatrix}, \qquad i = 1, 2, \cdots, k$$



$$\text{or} \quad \vec{X} = \begin{pmatrix} \vec{X}_1 & & & & \\ & \vec{X}_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \vec{X}_N \end{pmatrix}.$$

$\vec{X}_k$ is the (transverse) position of the k-th D-p brane.

Configuration space is $(\mathbf{R}^{9-p})^k/S_k$ due to permutation symmetry (Branes are identical-bosons).

If m of the branes are coincident $\vec{x} = \vec{x_1} = \vec{x_2} = \cdots = \vec{x_m}$ and

Example : N 0-branes

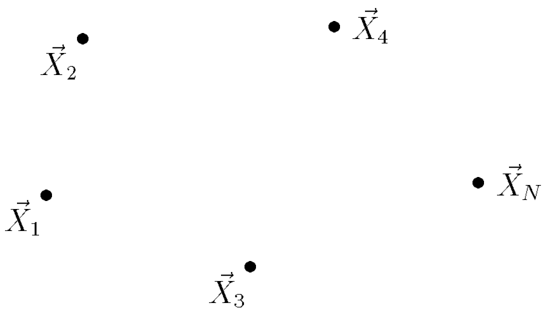
In gauge $A_0 = 0 A^a \to X^a, \quad a = 1, 2, \cdots, 9.$

$$L = \frac{1}{2g^2}\{\dot{X}^a \dot{X}_a + \sum_{a<b} \mathrm{tr}[X^a, X^b]^2 + 2\theta^T(\dot{\theta} + \Gamma_a[X^a, \theta])\}$$

θ : 16-component real spinor

Classical vacua $[X^a, X^b] = 0$

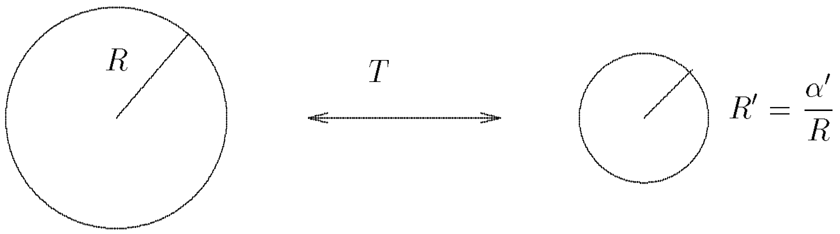
$$\vec{X} = \begin{pmatrix} \vec{X}_1 & & & & \\ & \vec{X}_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \vec{X}_N \end{pmatrix} \qquad : \text{ positions of the} N - 0\text{-branes}$$



Configuration space $= (\mathbf{R}^9)^N/S_N$.
Off-diagonal X^a give non-commutative geometry.

D-branes and T-Duality

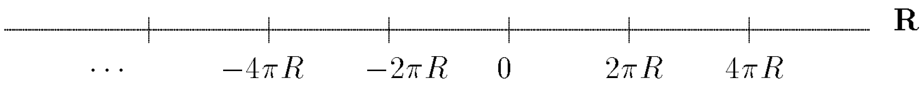
Consider strings on a circle of radius R giving $S^1 \times \mathbf{R}^9$



T-duality maps Neumann $\leftrightarrow$ Dirichelet.
Consider a D-p brane. Under T-duality we have

$$\text{p-brane} \begin{cases} \nearrow \ \text{p+1 brane if } S^1 \notin \text{p-brane} \\ \searrow \ \text{p-1 brane if } S^1 \in \text{p-brane} \end{cases}$$

How do we realize T-duality for Super-Yang-Mills? Consider for simplicity N 0-branes. What is the realization for $X^a, a = 1, 2, \cdots, 9$ for space $S^1 \times \mathbf{R}^9$? ($A_0 = 0$ gauge) We construct $S^1 = \mathbf{R}/2\pi R\mathbf{Z} = \mathbf{R}/\partial$.



The $N \times N$ matrices X^a are now realized as infinite dimensional matrices which are infinitely many copies of X^a in $\mathbf{R}^{10}$; we will quotient these matrices by Γ to obtain $S^1 \times \mathbf{R}^9 = \mathbf{R}^{10}/\Gamma$.

Organize the $\infty \times \infty$ matrices X^a into an infinite collection of $N \times N$ matrices each labelled by $m, n \in \mathbf{Z}$, i.e.

$$X^a \to \{X^a_{mn}\}$$

namely,

$$X^a = \begin{pmatrix} & \vdots & \vdots & \\ \cdots & X^a_{01} & X^a_{02} & \cdots \\ \cdots & X^a_{11} & X^a_{12} & \cdots \\ & \vdots & \vdots & \end{pmatrix}$$

Suppose X^1 corresponds to S^1. Then quotient by Γ leads to the following symmetry

$$\begin{aligned} X^i_{mn} &= X^i_{(m-1)(n-1)}, \quad i > 1 \\ X^1_{mn} &= X^1_{(m-1)(n-1)}, \quad m \neq n \\ X^1_{nn} &= 2\pi R I + X^1_{(n-1)(n-1)} \end{aligned}$$

From above it follows that all information is contained in the $N \times N$ matrices $X^a_{0n} \equiv X^a_n$ with $(X^a_n)^\dagger = X^a_n$. Using notation $X_k = X^1_{0k}$ we have

$$X^1 \sim M = \begin{pmatrix} \cdots & X_1 & X_2 & X_3 & \cdots & \\ X_{-1} & X_0 - 2\pi R & X_1 & X_2 & X_3 & \cdots \\ X_{-2} & X_{-1} & X_0 & X_1 & X_2 & \cdots \\ \cdots & X_{-2} & X_{-1} & X_0 + 2\pi R & X_1 & \cdots \end{pmatrix}$$

T-duality is now defined by the transformation

$$\begin{aligned} X^1 &\rightarrow A^1 = \sum_{n \in \mathbf{Z}} \mathrm{e}^{inx^1/R'} X^1_n \\ X^i &\rightarrow y^i = \sum_{n \in \mathbf{Z}} \mathrm{e}^{inx^1/R'} X^i_n, \quad \text{for } \ i > 1. \end{aligned}$$

and $M = i\partial^1 + A^1$ since by applying Fourier transformation to $i\partial^1$ we have

$$i\partial^1 = \mathrm{diag}(\cdots, -4\pi R, -2\pi R, 0, 2\pi R, 4\pi R, \cdots).$$

This is equivalent to the following for T-duality.

$$\begin{aligned} X^\alpha &\leftrightarrow (2\pi\alpha^1)(i\partial^\alpha + A^\alpha), \quad \alpha : \text{compact directions} \\ X^\beta &\leftrightarrow X^\beta, \qquad\qquad \beta : \text{noncompact directions} \end{aligned}$$

Example

$$A^1 = \frac{1}{2\pi R} \begin{pmatrix} \theta_1 & & 0 \\ & \cdot & \\ & & \cdot \\ 0 & & \theta_N \end{pmatrix}$$

Under T-duality

$$i\partial^1 + A^1 \rightarrow \frac{2\pi\alpha^1}{2\pi R} \begin{pmatrix} \theta_1 & & \\ & \cdot & \\ & & \cdot \\ & & & \theta_N \end{pmatrix} = X^1$$

That is $\partial^1 \equiv 0$ since this has been dualized and is inside the 1-brane and

$$X^1 = \text{position of the } ND1- \text{ branes} = \begin{pmatrix} \theta_1 R^1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \theta_N R^1 \end{pmatrix}$$

On taking the T-dual of the 0-brane we have obtained a 1-brane since the open string has DD-b.c's on dual S^1. Note

$$\begin{aligned} [X^1, X^a] &\rightarrow \int [i\partial^1 + A^1, X^a]^2 dx^1 \\ &= i \oint (\partial^1 X^a - i[A^1, X^a] \\ &= i \oint (D^1 X^a)^2 dx^1 \end{aligned}$$

$$
\begin{aligned}
(\partial^0 X^1)^2 &= \{\partial^0 X^1 - i[A^0, X^1]\}^2 \\
&= \oint \{\partial^0 (i\partial^1 + A^1) - i[A^0, i\partial^1 + A^1]\}^2 \\
&= \oint \{\partial^0 A^1 - \partial^1 A^0 - i[A^0, A^1]\}^2 \\
&= \oint (F_{01})^2 dx^1
\end{aligned}
$$

Hence by putting the prefactors in ($R^1 = \alpha/R$)

$$
\begin{aligned}
L &= \frac{\mathrm{tr}}{2g^2}(\dot{X}^1 \dot{X}_1 + ([X^1, X^a])^2 + \cdots) \\
&\rightarrow \frac{1}{R^1}\frac{1}{2g^2}\oint dx_1 \{\mathrm{tr} F_{01}^2 - (D^1 X^a)^2 + \cdots\}
\end{aligned}
$$

Hence the T-dual to 0-branes yields a $2D$ SYM with 16 supercharges.This result can be generalized to show that a D-p brane on $T^{|p-q|}$is T-dual to a D-q brane.We can also generalize the result to many compact dimensions. For directions $X^a \& X^b$ with radii $R_a \& R_b$ we have

$$
([X^a, X^b])^2 \rightarrow -\frac{1}{R'_a R'_b}\oint dx^a dx^b (F^{ab})^2
$$