Solution of PC1141 05-06

Physics Society

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1. $F_d = mgsin\theta$
   Static co-efficient friction = tan30^o = 0.577
   mgsinθ - f_k = ma
   Using s=ut+1/2 $\cdot$at^2 formulae,
   4.0m = $\frac{1}{2}$at^2
   $\rightarrow$a = 0.5m/s

   $\Rightarrow f_k = mgsin\theta - ma$
   = m(gsinθ-a)
   Kinetic co-efficient of friction = m(gsinθ - a)/mgsinθ
   $= \frac{gsin\theta - a}{gcos\theta}$
   $= \frac{9.81 \times sin30^o - 0.5}{9.81 \times cos30^o}$
   = 0.518

2. - Force to cause the oscillation = mgsinθ
     = ma

     sin$\theta \approx \theta$ for small angle
     $\rightarrow$ lθ = x

     $\Rightarrow$ ma = mg($\frac{x}{l}$) as sin$\theta \approx \theta$
     a = $-\frac{g}{l} \cdot x$
     $\omega^2 = \frac{g}{l}$
     $T = \frac{1}{2} \times (\frac{4\pi^2 l}{g})^{0.5} + \frac{1}{2} \times (\frac{4\pi^2 0.5}{g})^{0.5}$
     = **1.71s**
   - Note that the angular momentum is not conserved. Anyway conservation of energy still hold. Hence the highest height achieved by the pendulum is still **0.5m**.

3. - Rate of work done = Power
     V=$\frac{dx}{dt}$ = 15t^2-16t-30
     a = $\frac{d^2x}{dt^2} = 30t - 16$
     Power = Fv = 0.280kg $\times a \times V$
     At t = 2.0s, Power = -24.64W ignore negative as we would like to look at the magnitude
     At t = 4.0s, Power = 4251.5W

- Average Power $= \frac{1}{2}\int_{2.0}^{4.0} F \cdot dx$
  $= \frac{1}{2}\int_{2.0}^{4.0}(0.28) \cdot (30t - 16) \cdot (15t^2 - 16t - 30)dt$
  $= \frac{1}{2} \cdot 0.28\int_{2.0}^{4.0}(450t^3 - 480t^2 - 644t + 480)dt$
  $= 0.14 \cdot \left[\frac{450}{4}t^4 - \frac{720}{3}t^3 - 322t^2 + 480t\right]_2^4$
  =**1490W**

4. In one second, let m = mass of air strike the person.
   m = $1.5 \times 0.5 \times 150 \times \frac{1000}{3600} \times 1.2$kg
   = 37.5kg

   $\rightarrow$ Force = $\frac{mv}{t}$ = 37.5kg $\times 150 \times \frac{1000}{3600}$ m/s^2
   = **1562.5N**

5. - Given the wave equation is $y = 2.0cos(0.50\pi x - 200\pi t)$
     Therefore, amplitude = 2.0cm

     $0.50\pi = \frac{2\pi}{\lambda}$
     λ = 4cm

     $200\pi = 2\pi$f
     f = 100Hz

     T = $\frac{1}{f}$
     T = 0.01s

     Velocity = 100×0.04 = 4m/s

   - v = $\sqrt{\frac{T}{\mu}}$
     T = $v^2\mu$
     = (4m/s)$^2 \times \frac{0.005}{0.01}$
     = 8N

6. - Please refer to the text book for the show of inertia of a uniform cylinder where the symmetry axis passing through its center of mass.
   - G. Potential energy will convert into Kinetic energy + Elastic potential energy through out the process.
     $\Rightarrow 2.0 \times 9.81 \times 1 \times sin37^o = \frac{1}{2} \times 2.0 \times V^2 + \frac{1}{2} \times 20 \times (1.0)^2 + \frac{1}{2} \cdot (\frac{1}{2} \times 5 \times 0.3^2) \times (\frac{V}{0.3})^2$
     1.81J = $\frac{9}{4}V^2$kg
     V = 0.9m/s
   - Let h = the vertical distance where the block has slided down.
     $\rightarrow$ h = x sin37^o where x is the distance the block has slided down along the plane.

     $\Rightarrow$ mgxsin37^o = $0.5 \cdot 20 \cdot$ x^2

x = 1.18m (maximum distance)

- Let V = max velocity during the process.
  Let x = the point where max velocity achived
  $2 \times g \times \sin 37^o \cdot x = \frac{1}{2} \times 2.0V^2 + 0.5 \cdot (0.5 \times 5 \times 0.3^2) \cdot (\frac{V}{0.3})^2 + \frac{1}{2}20x^2$
  $11.8x = \frac{9}{4}V^2 + 10x^2$
  rearrange the terms, $\rightarrow \frac{2}{3} \cdot (11.8x - 10x^2)^{0.5} = V$
  $\frac{dv}{dx} = \frac{2}{3} \cdot 0.5 \cdot (11.8x - 10x^2)^{-0.5} \cdot (11.8 - 20x)$
  $\frac{dv}{dx}$=0 when x = 11.8/20 ≈ 0.59m
  V ≈ 1.24m/s at that point
- The total mechanical energy of the system is
  $\frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I(\frac{\dot{x}}{R})^2 - mgxsin\theta =$ constant
  Differentiate with respect to t, we get
  $kx\dot{x} + m\dot{x}\ddot{x} + \frac{1}{2}M_p\dot{x}\ddot{x} + mg\dot{x}sin\theta = 0$ (take note that M_p = mass of pulley)
  Eliminate the term $\dot{x}$, we have
  $kx + m\ddot{x} + \frac{1}{2}M_p\ddot{x} + mgsin\theta = 0$
  $\rightarrow (m + 0.5M_p)\ddot{x} = -kx - mgsin\theta$
  $= -k(x + \frac{mgsin\theta}{k})$
  Let $Y = (x + \frac{mgsin\theta}{k})$
  $\Rightarrow \ddot{Y}(m + 0.5M_p) = -kY$
  $\ddot{Y} = \frac{-k}{m+0.5M_p}Y$ (simple harmonic motion)

  Therefore, period $= \frac{1}{2\pi}\sqrt{\frac{m+0.5M_p}{k}}$
  $= \frac{1}{2}\sqrt{\frac{2.0+0.5\times5.0}{20}}$
  $= 0.075s$

7.
- $g = \frac{GM}{r^2}$
  $= G\frac{\frac{4}{3}\pi r^3\rho}{r^2}$
  $= \frac{4G\pi r\rho}{3}$
  = 6.64m/s

  speed of an object needed to escape from the gravitational pull = v
  $\frac{GMm}{r} = 0.5mv^2$
  $v = \sqrt{2 \times G \times \frac{4}{3}\pi r^2 \times \rho}$
  = 4778m/s
- Let x = center of mass of that planet
  Mass of the cavity $= \frac{4}{3} \cdot \pi \cdot (5400 \cdot 1000 \div 2)^3 \cdot \rho$
  $= 3.628 \times 10^{23}$kg
  Mass of the planet $= \frac{4}{3} \cdot \pi \cdot (5400 \cdot 1000)^3 \cdot \rho$ - 3.628×10^{23}kg
  $= 2.54 \times 10^{24}$kg

  (Center of mass of complete planet without cavity should be zero) $0 = \frac{2.54\times10^{24}\cdot x+3.628\times10^{23}\cdot 0.5\cdot 54}{2.54\times10^{24}+3.628\times10^{23}}$
  = -386km
- Escape velocity at point A $= \sqrt{\frac{2GM}{r}}$

$= \sqrt{\frac{2\times6.67\times10^{-11}\times2.54\times10^{24}}{(5400+386)\times1000}}$
$\approx$7650m/s

Escape velocity at point B = $\sqrt{\frac{2GM}{r}}$
$= \sqrt{\frac{2\times6.67\times10^{-11}\times2.54\times10^{24}}{(5400-386)\times1000}}$ = 8220m/s

- Escape velocity at point C is lower as compared to the time at point B as its distance from the center of mass of planet is longer.
  But it has higher velocity as compared to the time at point A. Using triangular inequality, we can show that its distance from the center of mass is shorter as compared to the distance from point A.
- Using $\frac{GM}{r^2}$, substitute point A distance from center of mass or point B distance from center of mass.
  We get g_B = 6.74m/s
  g_A = 5.06m/s

8.
- Resultant force results in circular motion
  mgsinθ - 110 = mv^2/r
  1.2 $\times$ 9.81 $\times$ sin40^o - 110 = -$\frac{1.2\times V^2}{0.75}$ (negative because toward the center point)
  $\rightarrow$ v = 8m/s
- The speed of the puck at highest point = V$_t$
  The tension of the string for highest point = T
  $\rightarrow$ T + mgsinθ = $\frac{mV_t^2}{r}$
  By conservation of energy
  = 0.5mV$_o^2$ = mgh + 0.5mV$_t^2$
  h = 0.75m $\times$ 2 $\times$ sin40^o and we have V$_o$ = 8m/s
  $\rightarrow$ V$_t$ = 6.71m/s
  Tension = 64.6N after substitution
- Using conservation of energy,
  Let V_0=minimum velocity of the puck required to complete circular path, and V_f=the velocity of the puck at highest point.
  $\frac{1}{2}mV_0^2 = \frac{1}{2}mV_f^2$ + mgh
  Note that h = 2L$sin\theta$
  At top, the tension in the string is zero for the case that the puck just almost complete the circular path.

  $\rightarrow \frac{mV_f^2}{r} = mgsin\theta$
  since r = L
  $\Rightarrow V_f = \sqrt{gLsin\theta}$
  substitute V_f inside the energy equation.
  $\frac{1}{2}mV_0^2 = \frac{1}{2}mgLsin\theta + mg2Lsin\theta$
  $\rightarrow V_0 = \sqrt{5gLsin\theta}$
  $= \sqrt{5 \times 9.81 \times sin40^o \times 0.75}$
  = 4.86m/s

Let T = Tension in the string when the puck has the minimum velocity at lowest position
T - $mgsin\theta = m(5gLsin\theta) \div L$
$\rightarrow T = 6mgsin\theta$
$= 6 \times 1.2 \times 9.81 \times sin40^o$
$= 45.4$N

- Let the velocity at lowest point required to complete the circular path $= V_0$ and the velocity at the highest point $= V_f$.
  T = tension of the string
  Based on the conservation of energy,
  $0.5mV_0^2 = 0.5mV_f^2 + mgh$
  $0.5mV_0^2 = 0.5mV_f^2 + mg(2Lsin\theta)$ (1)
  But $\frac{mV_f^2}{L} = T + mgsin\theta$
  Let's assume T = 0N when the puck is at highest point for minimum V_0.
  $\rightarrow \frac{mV_f^2}{L} = mgsin\theta$
  $V_f^2 = gLsin\theta$ subinto the equation (1)

  $\Rightarrow 0.5mV_0^2 = 0.5mgLsin\theta + mg2Lsin\theta$
  $V_0^2 = 5gLsin\theta$

  At last we have the tension T_0 at lowest point as $\frac{mV_0^2}{L} = T_0 - mgsin\theta$
  $T_0 = 6mgsin\theta$
  $= 6 \times 1.2 \times 9.81 \times sin40^o$
  $= 45.4$N
- the puck will slide following the circular path but it will not reach the top due to frictional force.
- this time the string has been replaced by a rod with negligible mass.
  Using conservation of energy again, but this time even when the puck reaches highest point, it can even complete the circular path even without velocity. (the rod will support puck but not the string when it is at highest point).
  $\frac{1}{2}mV_i^2 = mg2Lsin\theta$
  $V_i = \sqrt{4gLsin\theta}$
  $= \sqrt{4 \times 9.81 \times 0.75 \times sin40^o}$
  $= 4.35$m/s

- I think that even the rod has no negligibled mass, it will still need the same minimum velocity to complete the circular path. Because if we manage the give the rod with the same velocity as calculated in part (d), that means the rod has sufficient kinetic energy as well to complete the circular path.