

NUS PHYSICS SOCIETY

PAST YEAR PAPER SOLUTIONS

PC 1141 : AY 2012 /13

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1. An egg of mass 0.050 kg is in a stiff box of mass 0.300 kg, surrounded on all sides by padding that constrains the egg in place. A force of $F = (4.61\hat{i} + 2.80\hat{j})$ N acts on the outside of the box, in addition to the force of gravity (which is oriented in the $-y$ direction).

(a) Draw the free body diagrams for the egg-box system and for the egg. [4]

Approach

We first look at the forces acting on the egg-box system: there is the total weight of the egg and box acting on the system, and the force F acting on the outside of the box.

$$\begin{aligned}\vec{W}_{sys} &= (m_{egg} + m_{box})\vec{g} \\ &= (0.050 + 0.300)(-9.81\hat{j}) \\ &= -3.43\hat{j}N\end{aligned}$$

$$\vec{F}_{sys} = (4.61\hat{i} + 2.80\hat{j})$$

Next, we look at the forces acting on the egg: there is a force F_{egg} acting on the egg due to the resultant force from F on the egg-box system, and the weight of the egg.

$$\begin{aligned}\vec{W}_{egg} &= (m_{egg})\vec{g} \\ &= (0.050)(-9.81\hat{j}) \\ &= -0.49\hat{j}N\end{aligned}$$

We can now draw the free body diagrams:

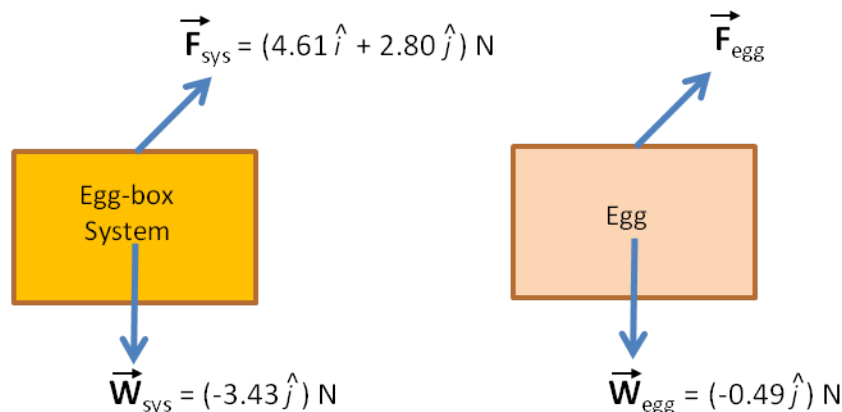


Figure 1

(b) What are the acceleration of the egg-box system and the net force on the egg? [4]

Approach

Firstly, we calculate the resultant force, $F_{res-sys}$, acting on the egg-box system. Then, using Newton's second law of $F = ma$, we can calculate acceleration of the system, a_{sys}

from the resultant force:

$$\begin{aligned}\vec{F}_{res-sys} &= \vec{F}_{sys} + \vec{W}_{sys} \\ &= (4.61\hat{i} + 2.80\hat{j}) + (-3.43\hat{j}) \\ &= (4.61\hat{i} - 0.63\hat{j})\end{aligned}$$

$$\begin{aligned}\vec{a}_{res-sys} &= \frac{\vec{F}_{res-sys}}{m_{sys}} \\ &= \frac{4.61\hat{i} - 0.63\hat{j}}{0.300 + 0.050} \\ &= (13.2\hat{i} - 1.8\hat{j}) N\end{aligned}$$

Next, since acceleration of the egg due to force F is equal to the acceleration of the system due to force F , we can determine F_{egg} using Newton's second law. We can then find the net force, $F_{res-egg}$, by adding the vector sum of the W_{egg} and F_{egg} . Then:

$$\begin{aligned}\vec{F}_{egg} &= \vec{a}_{egg}m_{egg} \\ &= \vec{a}_{sys}m_{egg} \\ &= \left(\frac{\vec{F}_{sys}}{m_{sys}}\right)m_{egg} \\ &= \left(\frac{4.61\hat{i} + 2.80\hat{j}}{0.300 + 0.050}\right)0.050 \\ &= (0.66\hat{i} + 0.40\hat{j}) N\end{aligned}$$

$$\begin{aligned}\vec{F}_{res-egg} &= \vec{F}_{egg} + \vec{W}_{egg} \\ &= (0.66\hat{i} + 0.40\hat{j}) + (-0.49\hat{j}) \\ &= (0.66\hat{i} - 0.09\hat{j})N\end{aligned}$$

2. The axis of the uniform cylinder shown in the figure below is fixed. The cylinder is initially at rest. The block of mass M is initially moving to the right without friction with speed v_1 . It passes over the cylinder to the dotted position. When it first makes contact with the cylinder, it slips on the cylinder, but friction is large enough so that the slipping ceases before M loses contact with the cylinder. Find the final speed of the block v_2 in terms of v_1 , M , radius of the cylinder R and mass of the cylinder m .

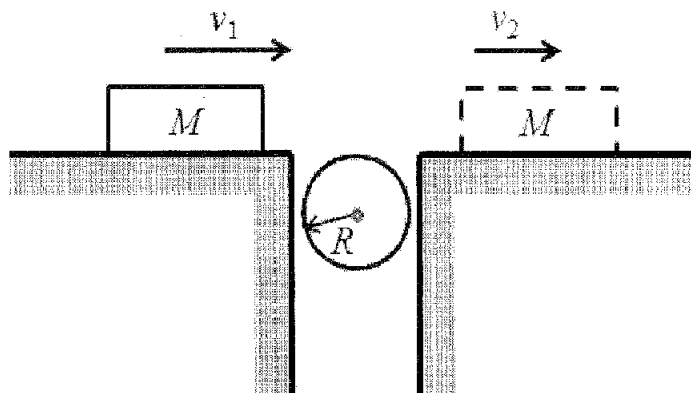


Figure 2

[8]

Approach

Recall that torque is the rate of change of angular momentum with respect to time.

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$fR\Delta t = I\omega$$

where

$$\omega = \frac{v^2}{R} \text{ and } I = \frac{1}{2}mR^2$$

$$\text{Torque: } fR = \frac{1}{2}mR^2\alpha$$

$$-f = ma$$

$$a = \frac{v_2 - v_1}{\Delta t}$$

$$-M\frac{v_2 - v_1}{\Delta t}R\Delta t = \frac{1}{2}mR^2\frac{v_2}{R}$$

$$2M(v_1 - v_2) = Mv_2$$

$$v_2 = \frac{2Mv_1}{2M + m}$$

3. A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown, so that the sides from a continuous plane, but the ends are offset at each block from the previous brick by a distance $0.15L$. how many bricks can be stacked in this manner before the pile topples over? [8]

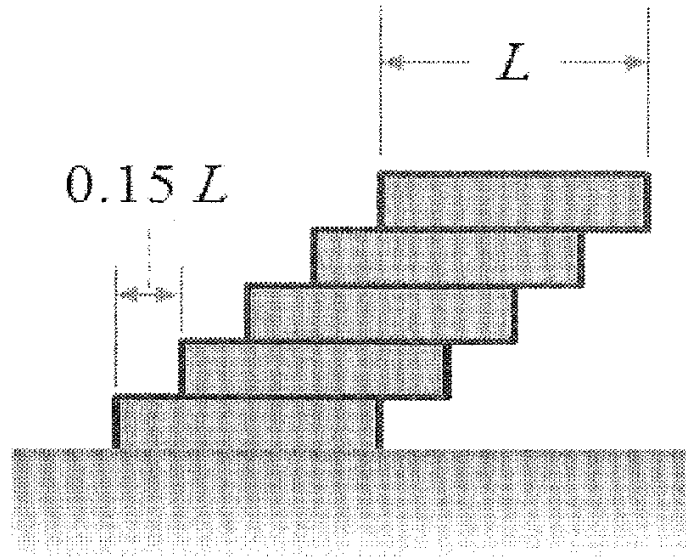


Figure 3

Approach

Centre of mass of each block is displaced a distance $0.15L$ to the right. Take origin at the left most edge of the bottom block. Note that x_{CM} must not exceed L .

$$\begin{aligned}
 x_{CM} &= \frac{\sum m_i x_i}{\sum m_i} \\
 &= \frac{\sum_{n=1}^N (0.5L + (n-1)0.15L)}{N \cdot m} \\
 &= \frac{N}{2N} (0.5L + 0.5L + (N-1)0.15L) \\
 &= \frac{1}{2} (L + (N-1)0.15L) \leq L
 \end{aligned}$$

$$1 + 0.15(N-1) \leq 2$$

$$N \leq 7\frac{2}{3}$$

$$\text{Max no. of blocks} = 7$$

4. A cylinder of density ρ_o , length L , and cross-section area A floats in a liquid of density ρ_f with its axis perpendicular to the surface. Length h of the cylinder is submerged when the cylinder floats at rest. Show, with clear explanation, that if the cylinder is displaced vertically slightly from its equilibrium position, it oscillates vertically with a period T given by

$$T = 2\pi\sqrt{\frac{h}{g}}.$$

[8]

Approach

Recall Archimedes' Principle for floatation:

$$\begin{aligned} \text{Weight of cylinder} &= \text{Buoyant force acting on cylinder} \\ \rho_o ALg &= \rho_f Ahg \\ -\rho_f A(h + h')g + \rho_o ALg &= \rho_o ALa \\ -\rho_f h'g &= \rho_f ha \\ a &= -\frac{g}{h}h' \end{aligned}$$

This tells us that the motion is simple harmonic motion, since acceleration is always opposite in direction to the displacement.

$$\begin{aligned} \omega^2 &= \frac{g}{h} \\ \frac{T^2}{4\pi^2} &= \frac{h}{g} \\ T &= 2\pi\sqrt{\frac{h}{g}} \end{aligned}$$

5. The left-hand end of a long, taut string is moved harmonically up and down with amplitude 0.050m. This motion produces a wave of wavelength $\lambda = 1.50\text{m}$ that travels in the $+x$ direction at speed $v=173\text{m/s}$.

(a) What is the frequency, f , of the travelling wave? [2]

Approach

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{173}{1.50} \\ &= 115.3\text{Hz} \end{aligned}$$

(b) What are the maximum transverse velocity and maximum transverse acceleration of a point along the string? [6]

Approach

$$\begin{aligned} \text{max transverse velocity} &= \omega A \\ &= 36.2 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{max transverse acceleration} &= \omega^2 A \\ &= 2.63 \cdot 10^4 \text{ ms}^{-2} \end{aligned}$$

6. Two blocks made of different materials and connected together by a thin cord, slide down a plane ramp inclined at an angle θ to the horizontal as shown in the diagram below (block 2 is above block 1). The masses of blocks 1 and 2 are m_1 and m_2 , and the coefficients of kinetic friction between the masses and the ramp are μ_1 and μ_2 respectively.

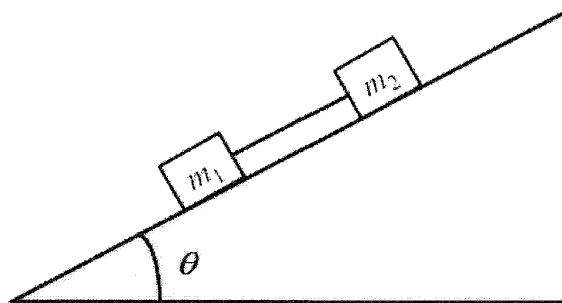


Figure 4

- (a) Briefly describe and explain the motion of the two blocks

- i. if $\mu_1 < \mu_2$ and

[3]

Approach

There are two accelerations acting on m_1 and m_2 parallel to the incline: acceleration due to gravity and acceleration in the opposite direction due to friction. Both of these types of accelerations are independent of the masses of the blocks. The acceleration experienced by both m_1 and m_2 due to gravity are equal, where the magnitude is $mg \cdot \sin\theta$. Hence, they should travel at the same speed when friction is not taken into account. However, if μ_1 is smaller than μ_2 , the acceleration due to friction for m_1 will also be smaller than the acceleration due to friction for m_2 , since the frictional accelerations are proportional to the coefficient of kinetic friction of each block as well as gravitational acceleration (which is constant for both block). m_1 should slide with faster acceleration than m_2 , but this results in the thin cord becoming taut. The taut cord exerts a tension on both blocks, such that the net accelerations a_1 and a_2 of the two blocks will be equal in magnitude and direction to each other. Both blocks will move with the same motion with the cord being taut between them throughout.

- ii. if $\mu_1 > \mu_2$

[3]

Approach

There are two accelerations acting on m_1 and m_2 parallel to the incline: acceleration due to gravity and acceleration in the opposite direction due to friction. Both of these types of accelerations are independent of the masses of the blocks. The acceleration experienced by both m_1 and m_2 due to gravity are equal, where the magnitude is $mg \cdot \sin\theta$. Hence, they should travel at the same speed when friction is not taken into account. However, if μ_1 is greater than μ_2 , the acceleration due to friction for m_1 will also be greater than the acceleration due to friction for m_2 , since

the frictional accelerations are proportional to the coefficient of kinetic friction of each block as well as gravitational acceleration (which is constant for both block). Hence, the net acceleration, a_1 , of m_1 will be smaller than the net acceleration, a_2 , of m_2 . m_2 will slide with faster acceleration than m_1 and the thin cord will go slack. Given enough sliding distance, m_2 will catch up with m_1 and they will coalesce and slide down with the same speed and acceleration as each other.

- (b) Determine the expressions for the accelerations, a_1 and a_2 , of the blocks and tension T in the cord for both cases above. Simplify your expressions as much as possible. [9]

Approach

For the case of $\mu_1 > \mu_2$:

Since acceleration of m_1 is the same as that for m_2 , let a be the net acceleration of each block.

Consider forces acting on m_1 ,

$$g \sin \theta - \mu_1 m_1 g \cos \theta - T = m_1 a$$

Consider forces acting on m_2 ,

$$m_2 g \sin \theta - \mu_2 m_2 g \cos \theta + T = m_2 a$$

Combining the two,

$$\begin{aligned} (m_1 + m_2)a &= g \sin \theta (m_1 + m_2) - g \cos \theta (\mu_1 m_1 + \mu_2 m_2) \\ a &= \frac{g \sin \theta (m_1 + m_2) - g \cos \theta (\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \\ a &= g \left[\sin \theta - \frac{\cos \theta (\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \right] \end{aligned}$$

Now we can find tension, T , by substituting a :

$$\begin{aligned} T &= m_2 a - m_2 g \sin \theta + \mu_2 m_2 g \cos \theta \\ T &= m_2 g \left[\sin \theta - \frac{\cos \theta (\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \right] - m_2 g \sin \theta + \mu_2 m_2 g \cos \theta \\ &= m_2 [(m_1 + m_2) g \sin \theta - \cos \theta (\mu_1 m_1 + \mu_2 m_2)] \\ &\quad - (m_1 + m_2) m_2 g \sin \theta + (m_1 + m_2) \mu_2 m_2 g \cos \theta \\ &= -(\mu_1 m_1 + \mu_2 m_2) m_2 g \cos \theta + (m_1 \mu_2 + m_2 \mu_2) m_2 g \cos \theta \\ &= (\mu_2 - \mu_1) m_1 m_2 g \cos \theta \\ T &= \frac{(\mu_2 - \mu_1) m_1 m_2 g \cos \theta}{m_1 + m_2} \end{aligned}$$

For the case of $\mu_1 < \mu_2$:

We know that there is no tension in the cord. We can thus directly calculate acceleration for m_1 and m_2

$$\begin{aligned} T &= 0 \\ a_1 &= g \sin \theta - \mu_1 g \cos \theta \\ &= g(\sin \theta - \mu_1 \cos \theta) \\ a_2 &= g \sin \theta - \mu_2 g \cos \theta \\ &= g(\sin \theta - \mu_2 \cos \theta) \end{aligned}$$

- (c) Explain how the answers to parts (a) and (b) would change (if at all), if the two blocks sliding on the plane are connected by a light rigid rod instead of a cord. [5]

Approach

In the case of ($\mu_1 < \mu_2$), the light rigid rod has the same effect as a string and the tension in the rod will be the same as that for the string. Since the net forces acting on the two blocks would be the same, the answers to part (a) and (b) would have no change.

However, for the case of ($\mu_1 > \mu_2$), the light rod provides a tension acting on the the two blocks. This is in contrast to the string, in which there are no forces acting on the two blocks by the string. Hence, we would expects the answers to change for both part (a) and (b).

For part (a), without the light rigid rod, the net acceleration, a_1 , of m_1 will be smaller than the net acceleration, a_2 , of m_2 . However, due to the presence of a light rod, the light rod will apply a force on both blocks in a way such that their accelerations will be the same. In other words, due to the light rigid rod connecting the two blocks, we can see the two block and the rod as one object, moving with the same velocity and acceleration.

For part (b), Since acceleration of m_1 is the same as that for m_2 , let a be the net acceleration of each block.

Consider forces acting on m_1 ,

$$m_1 g \sin \theta - \mu_1 m_1 g \cos \theta + T = m_1 a$$

Consider forces acting on m_2 ,

$$m_2 g \sin \theta - \mu_2 m_2 g \cos \theta - T = m_2 a$$

Combining the two,

$$\begin{aligned} (m_1 + m_2)a &= g \sin \theta (m_1 + m_2) - g \cos \theta (\mu_1 m_1 + \mu_2 m_2) \\ a &= \frac{g \sin \theta (m_1 + m_2) - g \cos \theta (\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \\ a &= g \left[\sin \theta - \frac{\cos \theta (\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \right] \end{aligned}$$

Now we can find tension, T , by substituting a :

$$\begin{aligned}
 -T &= m_2 a - m_2 g \sin \theta + \mu_2 m_2 g \cos \theta \\
 -T &= m_2 g \left[\sin \theta - \frac{\cos \theta (\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \right] - m_2 g \sin \theta + \mu_2 m_2 g \cos \theta \\
 &= m_2 [(m_1 + m_2) g \sin \theta - \cos \theta (\mu_1 m_1 + \mu_2 m_2)] \\
 &\quad - (m_1 + m_2) m_2 g \sin \theta + (m_1 + m_2) \mu_2 m_2 g \cos \theta \\
 &= -(\mu_1 m_1 + \mu_2 m_2) m_2 g \cos \theta + (m_1 \mu_2 + m_2 \mu_2) m_2 g \cos \theta \\
 &= (\mu_2 - \mu_1) m_1 m_2 g \cos \theta \\
 -T &= \frac{(\mu_2 - \mu_1) m_1 m_2 g \cos \theta}{m_1 + m_2} \\
 T &= \frac{(\mu_1 - \mu_2) m_1 m_2 g \cos \theta}{m_1 + m_2}
 \end{aligned}$$

7. (a) A uniform solid ball of radius r_0 rolls on the inside of a track of radius R_0 (see figure below). If the ball starts from rest at the vertical edge of the track, what will its speed be when it reaches the lowest point of the track. Assume that the ball always rolls without slipping. [5]

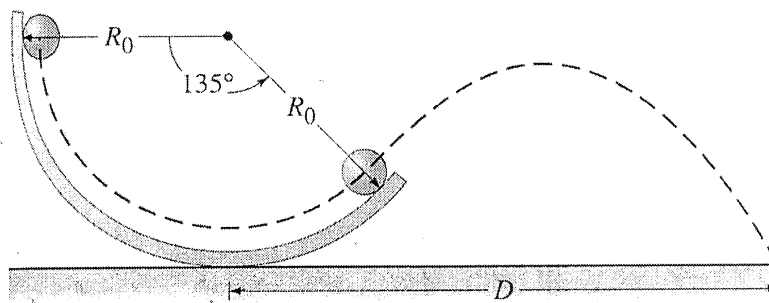


Figure 5

Approach

We first apply the Conservation of Energy:

$$\begin{aligned}
 mg(R_0 - r_0) &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
 g(R_0 - r_0) &= \frac{1}{2}v^2 + \frac{1}{5}v^2 \\
 v_{\text{bottom}} &= \sqrt{\frac{10}{7}g(R_0 - r_0)}
 \end{aligned}$$

- (b) For this part of the question, take $r_0 = 1.5\text{cm}$ and $R_0 = 26.0\text{cm}$ and the ball starts rolling at height r_0 above the bottom of the track. It leaves the track after passing through 135° as shown. [9]
- i. What will its speed be when it leaves the track?

Approach

Let $\theta = 45^\circ$ be the angle between the lowest point of the track to the end of the track. Again, we will apply the Conservation of Energy:

$$\begin{aligned}
 mg(R_0 - r_0) \cos \theta &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
 g(R_0 - r_0) \cos \theta &= \frac{1}{2}v^2 + \frac{1}{5}v^2 \\
 v_0 &= \sqrt{\frac{10}{7}g(R_0 - r_0) \cos \theta}
 \end{aligned}$$

- ii. At what distance D from the base of the track will the ball hit the ground?

Approach

We first find the height, h_0 of the center of mass of the ball at the end of the track.

$$\begin{aligned} h_0 &= R_0 - (R_0 - r_0) \cos \theta \\ &= R_0 (1 - \cos \theta) + r_0 \cos \theta \end{aligned}$$

We then find the height difference, H , of the centre of mass of ball from the end of the track to the ground.

$$\begin{aligned} H &= R_0 - (R_0 - r_0) \cos \theta - r_0 \\ &= R_0 (1 - \cos \theta) + r_0 (\cos \theta - 1) \\ &= (R_0 - r_0) (1 - \cos \theta) \end{aligned}$$

Using kinematics, we can then solve for the question.

$$\begin{aligned} v_0 t \sin \theta - \frac{1}{2} g t^2 &= -H \\ \frac{1}{2} g t^2 - v_0 t \sin \theta - H &= 0 \\ t &= \frac{v_0 \sin \theta \pm \sqrt{v_0^2 \sin^2 \theta + 2gH}}{g} \end{aligned}$$

Taking positive root of t ,

$$\begin{aligned} t &= \frac{v_0 \sin \theta}{g} + \frac{\sqrt{v_0^2 \sin^2 \theta + 2gH}}{g} \\ \text{Range} &= v_0 \cos \theta t + (R_0 - r_0) \sin \theta \\ D &= \frac{v_0^2 \sin(2\theta)}{2g} \frac{t}{v_0 \cos \theta} + g \sqrt{v_0^2 \sin^2 \theta + 2gH} \\ &= 0.480m \end{aligned}$$

- (c) Explain how the results to parts (a) and (b) would change (if at all) when the ball is replaced by another ball of same radius and mass but is hollow, i.e., consider a spherical shell. [6]

Approach

For a hollow sphere, its moment of inertia, I , will be larger than that of the original ball. Although all the gravitational potential energy is converted into kinetic energy, the velocity component of the kinetic energy will be smaller than that of the ball, due to its greater moment of inertia. Hence, the velocity at the lowest part of the track in part (a) would be smaller, and the distance in (b) would also be smaller since it has a smaller horizontal velocity when leaving the track.

8. Galileo discovered the four largest moons of Jupiter in 1609. the table below gives the mass of the moons, period of revolution around Jupiter and the mean distance of these moons from Jupiter (measured from the centre of mass of Jupiter to the centre of mass of the moons).

Moon	Mass (kg)	Period (Earth Days)	Mean distance (km)
Io	8.9×10^{22}	1.77	422×10^3
Europa	4.9×10^{22}	3.55	671×10^3
Ganymede	1.5×10^{23}	7.16	1070×10^3
Callisto	1.1×10^{23}	16.7	1883×10^3

Figure 6

- (a) From the above data, determine the mass of Jupiter. (You may assume circular orbit with radius given by the mean distance and you need not work out the uncertainty.) [6]

Approach

$$\begin{aligned}
 T^2 &= \frac{4\pi^2}{GM_{Jupiter}} r^3 \\
 M_{Jupiter} &= \frac{4\pi^2}{GT^2} r^3 \\
 &= 1.9 \times 10^{27} \text{ kg}
 \end{aligned}$$

- (b) The radius of Io is $1.82 \times 10^3 \text{ km}$. On the surface of Io, what is the value of g , i.e., the acceleration due to gravitational pull of Io itself? [2]

Approach

$$\begin{aligned}
 g &= \frac{GM}{R^2} \\
 &= 1.79 \text{ m s}^{-1}
 \end{aligned}$$

- (c) A man is on the surface of Io and Jupiter is directly overhead.
- What is the magnitude of the force on a ball of mass m due to the gravitation attraction of Jupiter? How is this value compared to that due to Io?

Approach

$$\begin{aligned}
 F_{Jupiter} &= \frac{GM_{Jupiter} m_{ball}}{R_{Jupiter-Io}^2} \\
 &= 0.71 \text{ N}
 \end{aligned}$$

- ii. When the man drops the ball on the surface of Io, would the ball accelerate with g as determined in (b) above or would it be substantially lesser due to the attractive pull of Jupiter. Explain your answer.

[6]

Approach

$$\frac{F_{Jupiter}}{F_{Io}} = 0.40$$

- (d) What is the energy needed to totally disintegrate the moon Io, i.e. to increase the distance between all individual molecules of Io to infinity? Neglect all other masses around Io and consider only gravitational effects.

[6]

Approach

$$\begin{aligned} dU &= -\frac{GmdM}{R} \\ dM &= 4\pi r^2 dr \rho \\ M &= \frac{4}{3}\pi r^3 \rho \\ U &= -\frac{16G\rho^2\pi^2}{3} \int_0^R r^4 dr \\ &= -\frac{16GM^2\pi^2}{3^{\frac{4}{3}}\pi^2 R^3} \frac{R^5}{5} \\ &= -\frac{3GM}{5} \frac{GM}{R} \\ \text{Energy Required} &= \frac{3GM}{5R} \end{aligned}$$