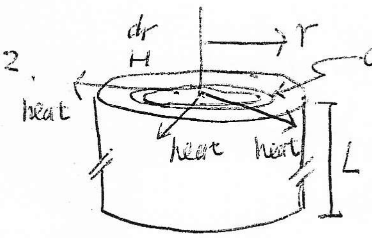
PC1142 AY2011

1. $H_2O(l) \rightarrow H_2O(g)$ $\Delta H_{vap} = 40.6\ kJ\ mol^{-1}$

$$\Delta S_{vap} = \frac{\Delta H_{vap}}{T} = \frac{40.6\ kJ\ mol^{-1}}{373\ K} = 109\ J K^{-1} mol^{-1}$$
$$= (109\ J K^{-1} mol^{-1})\left(\frac{1}{0.018\ kg\ mol^{-1}}\right)$$
$$= 6.06\ kJ\ K^{-1}\ kg^{-1}$$

2.



Consider a cylindrical shell with radius r and width dr. Heat power produced in the volume enclosed by shell is

$$P = \pi r^2 L \cdot p \qquad \text{Eq (1)}$$

This heat has to be conducted away through the shell. At steady-state

$$P = -k(2\pi r L)\frac{dT}{dr} \qquad \text{Eq (2)}$$

Equating (1) and (2) and rearranging:

$$-\frac{dT}{dr} = \frac{rp}{2k}$$

Separate variables and integrate with boundary conditions

@ $r=R$, $T=T_{rim}$

@ $r=r$, $T=T$

$$\int_T^{T_{rim}} -dT = \int_r^R \frac{p}{2k} r\,dr$$

$$T - T_{rim} = \frac{p}{4k}(R^2 - r^2)$$

3. $$q = \sum_I g_I \exp\left(-\frac{\varepsilon_I}{k_B T}\right)$$

For the molecule $\varepsilon_I = \{n\varepsilon\}$ where n is an integer ≥ 0

$$q = \sum_{n=0}^{\infty} \exp\left(-\frac{n\varepsilon}{k_B T}\right) = 1 + \exp\left(\frac{-\varepsilon}{k_B T}\right) + \exp\left(\frac{-2\varepsilon}{k_B T}\right) + \cdots = 1 + \exp\left(-\frac{\varepsilon}{k_B T}\right) + \left[\exp\left(-\frac{\varepsilon}{k_B T}\right)\right]^2 + \cdots$$
$$= \frac{1}{1 - \exp\left(-\frac{\varepsilon}{k_B T}\right)}$$

Probability that the molecule occupies Level with $\varepsilon_I = \varepsilon$ is $\exp\left(\frac{-\varepsilon}{k_B T}\right)\left(1 - \exp\left(-\frac{\varepsilon}{k_B T}\right)\right)$

4. work done on the gas by left piston $W_1 = -\int p\,dV = p_1V_1$ (isobaric process)

work done on the gas by right piston $W_2 = +\int p\,dV = -p_2V_2$

Total work done on the gas $W = W_1 + W_2 = p_1V_1 - p_2V_2$

According to 1st law of thermodynamics

$$\Delta U = W + Q$$

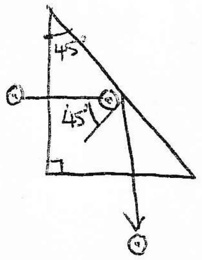
Since process is adiabatic, $Q = 0$, so $\Delta U = W$,

Thus $U_2 - U_1 = p_1V_1 - p_2V_2 \Rightarrow U_2 + p_2V_2 = U_1 + p_1V_1$

$$H_2 = H_1$$

so this process is isoenthalpic

5.

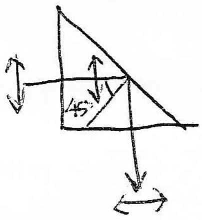


Consider the e-wave:

$\sin\theta_c = \dfrac{1}{1.486}$

which gives $\theta_c = 42.3°$

angle of incidence is $> \theta_c$, hence the e-wave is totally internally reflected

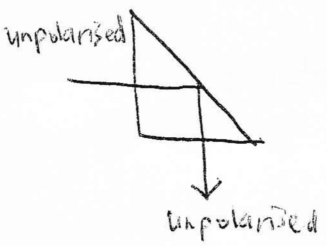


Consider the o-wave:

$\sin\theta_c = \dfrac{1}{1.658}$

which gives $\theta_c = 37.1°$

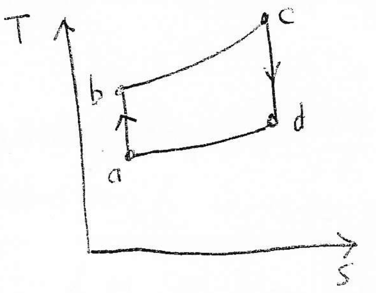
angle of incidence is $> \theta_c$, hence the o-wave is also totally internally reflected.



The light will emerge from the bottom face unpolarised

6.

(i)



For isochoric heating $b \to c$:

$$\Delta S = \int \frac{dq_{rev}}{T} = \int_b^c n\, C_v \frac{dT}{T} = n C_v \ln\left(\frac{T_c}{T_b}\right)$$

For isobaric cooling $d \to a$:

$$\Delta S = \int \frac{dq_{rev}}{T} = \int_d^a n C_p \frac{dT}{T} = n C_p \ln\left(\frac{T_a}{T_d}\right)$$

(ii) Heat input occurs over $b \to c$: $q_H = n\, C_v\, (T_c - T_b)$ (isochoric heating)

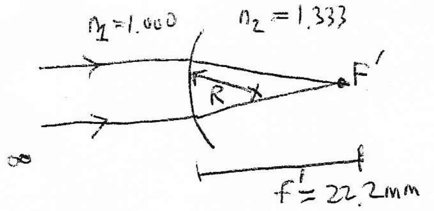
Heat output occurs over $d \to a$: $q_C = n\, C_p\, (T_d - T_a)$ (isobaric cooling)

Thermal efficiency $e_{th} = 1 - \frac{q_C}{q_H} = 1 - \frac{C_p (T_d - T_a)}{C_v (T_c - T_b)} = 1 - \gamma \frac{(T_d - T_a)}{(T_c - T_b)}$

To improve efficiency increase, T_c and/or decrease T_b, decrease T_d and T_a

Increase the length of the adiabatic segment $c \to d$, shorten the length of the adiabatic segment $a \to b$

7. (i)



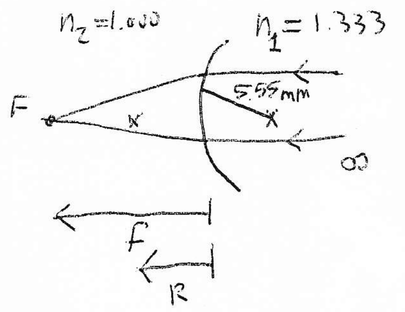
Spherical refracting surface equation:

$$\frac{n_2}{f'} = \frac{n_2 - n_1}{R}$$

$$R = \frac{n_2 - n_1}{n_2} \cdot f' = \frac{(1.333 - 1.000)}{1.333} (22.2\text{mm})$$

$$= 5.55 \text{ mm}$$

(ii)



Spherical refracting surface equation

$$\frac{n_2}{f} = \frac{n_2 - n_1}{R}$$

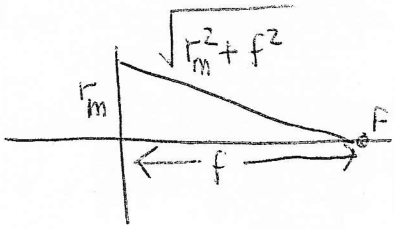
$$f = \frac{R n_2}{n_2 - n_1} = \frac{(-5.55\text{mm})(1.000)}{1.000 - 1.333} = 16.7\text{mm}$$

(iii) Angular half width of disk $\theta = \frac{1.22\lambda}{a n} = \frac{(1.22)(550\text{nm})}{(3.0\text{mm})(1.333)} = 1.7 \times 10^{-4}$ rad

Diameter of disk $= 2f'\theta = 2(22.2\text{mm})(1.7 \times 10^{-4}\text{rad})$

$= 7.5\ \mu\text{m}$

8. (i)



$\sqrt{r_m^2+f^2} - f = \delta_m$ optical path difference

If $\delta_m = \frac{m\lambda}{2}$, the path difference between waves successive zones is λ, so constructive interference

$$\sqrt{r_m^2+f^2} - f = \frac{m}{2}\lambda \qquad m \in \text{odd}$$

$$r_m^2 + f^2 = (f + \tfrac{m}{2}\lambda)^2$$

$$r_m = \sqrt{m\lambda f + \tfrac{1}{4}m^2\lambda^2}$$

(ii) $r_m \cong \sqrt{m\lambda f}$ for $\frac{m\lambda}{4} << f$

Take derivatives of both sides,

$$0 = \frac{1}{2}\frac{1}{\sqrt{m\lambda f}}\left(m\lambda df + mf d\lambda\right)$$

Hence

$$m\lambda df + mf d\lambda = 0$$

$$\frac{df}{d\lambda} = \frac{-f}{\lambda}$$