NATIONAL UNIVERSITY OF SINGAPORE

PC1142 Physics II

Semester I: AY 2011-12

25 November 2011

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper comprises **EIGHT (8)** printed pages with **FIVE (5) short questions** in Part 1 and **THREE (3) long questions** in Part 2. The total marks for Part 1 is 40 and Part 2 is 60. Please check you have all the pages.
2. A list of physical constants and formulae is given on pages 2 and 3.
3. Please answer **ALL** questions.
4. Please write all answers in the answer booklet provided.
5. You may use electronic calculators.
6. This is a **CLOSED BOOK** examination.

Thermal physics and kinetic theory:

(i) Kinetic mean free path is given by $\ell = \dfrac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\dfrac{3k_BT}{m}}$.

(b) Average speed: $v_{av} = \sqrt{\dfrac{8k_BT}{\pi m}}$.

(c) Most-probable speed: $v_{mp} = \sqrt{\dfrac{2k_BT}{m}}$.

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$.

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T.$$

(v) One form of adiabat: $p \cdot V^{\gamma} = constant$.

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$.

(vii) Planck radiation equation:

$$P_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)}.$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3} m \cdot K}{T}.$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$.

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \dfrac{\Delta T}{L}$.

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$.

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$.

(ii) Carnot heat-engine efficiency $e_C = 1 - \dfrac{T_C}{T_H}$.

(iii) Entropy $dS = \dfrac{dq_{rev}}{T}$.

(iv) Enthalpy $H = E + PV$.

(v) Helmholtz free energy $F = E - TS$.

(vi) Gibbs free energy $G = H - TS$.

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$,

where $\dfrac{1}{f} = (n-1) \cdot \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$.

(b) Refracting-surface equation: $\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$.

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \dfrac{d}{n}$.

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}}.$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}}.$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$.

(iv) F-number: $\frac{F}{\#}=\frac{f}{D}$.

(v) Numerical aperture: $NA = n \cdot \sin\theta$.

(vi) Snell's law: $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$.

(a) Critical angle: $\sin\theta_c = n_2 / n_1$.

(b) Brewster angle: $\tan\theta_p = n_2 / n_1$.

(vii) Wave relation: $v = f \cdot \lambda$.

(viii) Abbe number: $v = \frac{n_D - 1}{n_F - n_C}$.

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta = \frac{1.22\lambda}{a}$.

(ii) Slit: first diffraction minimum is at $\sin\theta = \frac{\lambda}{a}$.

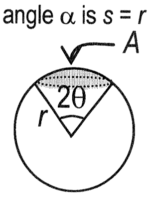
(iii) N-slit intensity pattern: $I = I_o \cdot \frac{\sin^2(N \cdot \phi/2)}{\sin^2(\phi/2)}$, where $\phi = \frac{2\pi}{\lambda} \cdot d \cdot \sin\theta$.

(iv) Single-slit diffraction pattern:

$I = I_o \cdot \frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta$.

General:

(i) Geometry:
The arc length on a circle of radius r subtended by angle α is $s = r\,\alpha$.



The surface area A subtended by polar angle 2θ is:

$A = 2\,\pi r^2 (1 - \cos\theta)$.

(ii) Logarithms and exponents :

$\log_a (b\,c) = \log_a b + \log_a c$

$\log_a b = \log_d b / \log_d a$

$a^b * a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

(iii) Integrations:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ except for $n = -1$ for which $\int x^{-1} dx = \ln x + c$.

(iv) Taylor expansions:

$\sin\theta = \theta - \frac{1}{6}\theta^3 + ...$

$\cos\theta = 1 - \frac{1}{2}\theta^2 + ...$

$\tan\theta = \theta + \frac{1}{3}\theta^3 + ...$

$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + ...$

(v) Series sum:

$\sum_{n=0}^{n=m} a_0 r^n = \frac{a_o(1-r^{m+1})}{1-r}$

Universal constants:

Gas constant $R = 8.314$ J K^{-1} mol^{-1}

Boltzmann constant $k_B = 1.381\text{x}10^{-23}$ J K^{-1}

Stefan–Boltzmann constant $\sigma = 5.670\text{x}10^{-8}$ W m^{-2}K^{-4}

Speed of light in vacuum $c = 2.998\text{x}10^{8}$ m s^{-1}

Avogadro's number $N_A = 6.022\text{x}10^{23}$ mol^{-1}

Permittivity of free space $\varepsilon_o = 8.854\text{x}10^{-12}$ Fm^{-1}

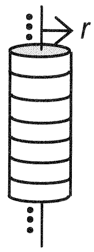
Planck constant $h = 6.626\text{x}10^{-34}$ J s

PC1142 Physics II

Part 1 Answer all **FIVE** questions. All questions carry 8 marks each.

1. The enthalpy of vaporization of water at its normal boiling temperature of 100°C is 40.6 kJ mol^{-1}. Compute its specific entropy of vaporization (i.e., per unit mass of water).

2. A typical nuclear fuel rod contains circular uranium oxide (UO_2) fuel pellets 10 mm in diameter and 5-mm thick stacked in a column to a length of 4 m inside a thin zirconium alloy tube, as shown below. The pellets generate heat uniformly throughout their volume due to nuclear fission, with a power density ρ (i.e., the heat power produced per unit volume of the pellet) that depends on their ^{235}U enrichment. This heats up the water in the reactor to produce steam to drive the turbine. Assuming that the rim of the fuel pellet is maintained at a constant temperature T_{rim} due to water cooling, show that the steady-state temperature profile $T(r)$, where r is the radial distance from the centre of the pellet and fuel rod, is given by: $T(r) = T_{rim} + \dfrac{\rho(R^2 - r^2)}{4k}$, where k is the thermal conductivity of the pellet and R is its radius.

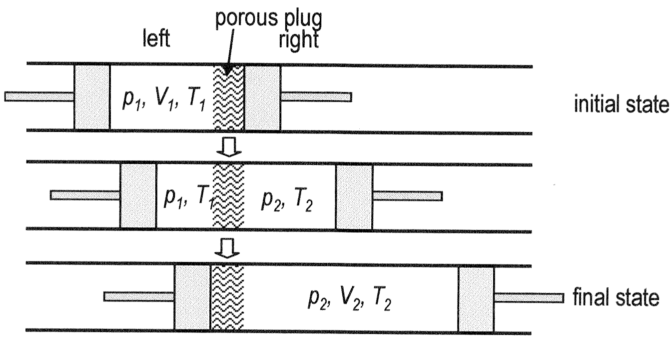


partial stacked column of fuel pellets in rod

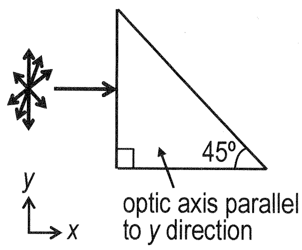
3. In statistical mechanics, the molecular partition function q gives the number of states that are thermally accessible to a collection of molecules at temperature T: $q = \sum_{I} g_I \exp(-\frac{\varepsilon_I}{k_B T})$, where ε_I is the energy of level I, g_I is its degeneracy, k_B is the Boltzmann constant, and the sum is taken over all levels I. Consider a molecule with an infinite number of equally spaced and non-degenerate energy levels, with ε_I = 0, ε, 2ε, 3ε, ..., where ε is a positive constant. Write an expression for the q of this molecule.

Hence or otherwise, write an expression for the probability that this molecule occupies the energy level with $\varepsilon_I = \varepsilon$.

4. Consider a gas being expanded *adiabatically* through a porous plug which separates a region with constant pressure p_1 and temperature T_1 (on the left, as shown below) from a second region with constant pressure p_2 and temperature T_2 (on the right). The gas may not be an ideal gas. Write an expression for the work done on the gas w_1 by the left piston if a volume V_1 crosses the porous plug. Then write an expression for the work done on this gas w_2 by the right piston. Note that the volume of the gas that emerges on the right hand side becomes V_2. Using the first law of thermodynamics or otherwise, show that this process is isoenthalpic (i.e., $\Delta H = 0$).



5. An unpolarised light beam is incident normally on a 45°-right-angled birefringent calcite prism that is oriented as shown below. Calcite has an extraordinary refractive index n_e of 1.486, and an ordinary refractive index n_o of 1.658. Note that n_e applies to light with electric field polarisation that is parallel to the optic axis. Compute and sketch on a diagram the direction of travel of the beam and give its state of polarization when it emerges outside the prism.



Part 2 Answer all **THREE** questions. All questions carry 20 marks each.

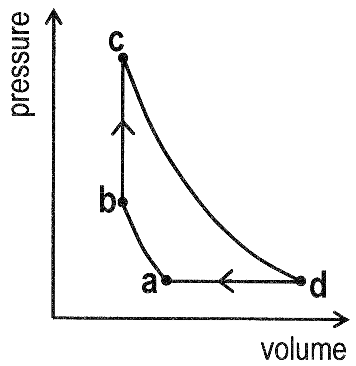
6. The Toyota Prius hybrid electric vehicle engine uses the Atkinson cycle. This cycle is able to achieve a higher thermal efficiency than the Otto cycle for given temperature limits. The Aktinson cycle comprises four steps as shown below:

a*→*b: adiabatic compression of air;

b*→*c: isochoric heating due to fuel combustion;

c*→*d: adiabatic expansion of the hot gas (power stroke);

d*→*a: isobaric cooling (exhaust).

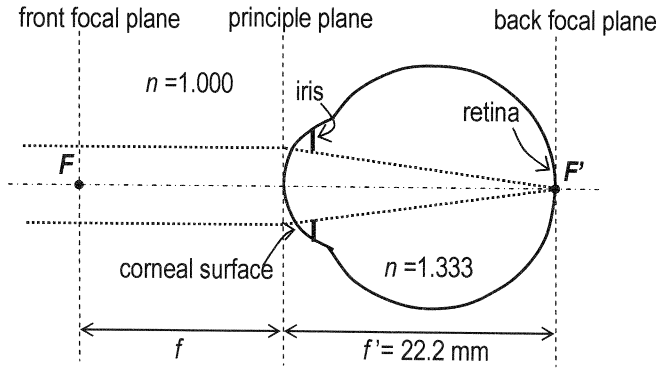


(i) Sketch the temperature *vs* entropy (*T–S*) state diagram for this process. Label the diagram with the points ***a***, ***b***, ***c*** and ***d***, and the direction of the cycle. Write the equations for the dependence of ΔS on temperature for ***b*→*c*** and ***d*→*a***.

[10 marks]

(ii) Derive an expression for the thermal efficiency of this cycle in terms of the temperatures T_a, T_b, T_c and T_d, and the heat capacity ratio of the working gas γ which is assumed to be a constant. Hence or otherwise, explain how one may improve the thermal efficiency of this cycle. [10 marks]

7. The Emsley 60-Diopter reduced eye model which is shown below is the simplest model of the relaxed human eye. It comprises a single spherical refracting surface (called the corneal surface) to model the behaviour of the eye. The focusing power of this corneal surface represents the combined powers of both the cornea and eye lens, while the eye interior is assumed to have a refractive index *n* of 1.333. The relaxed eye focuses objects at infinity onto the retina, which is positioned 22.2 mm from the corneal surface along the optical axis. The entrance pupil is controlled by the iris.

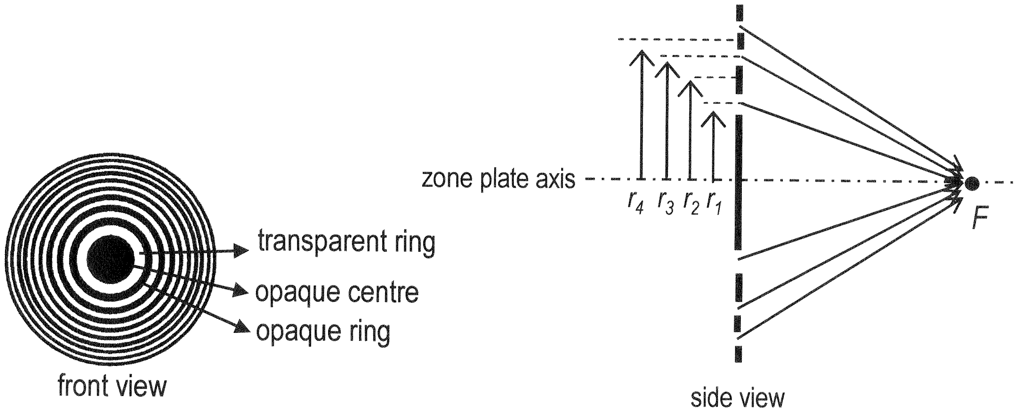


(i) Compute the radius of curvature of the corneal surface. [6 marks]

(ii) Hence or otherwise, compute the front focal length *f* in air. [6 marks]

(iii) If the entrance pupil diameter is 3.0 mm on a cloudy day, compute the diameter of the Airy disk on the retina given by its first dark ring for green light (vacuum wavelength, 550 nm). [8 marks]

8. A zone plate can focus x-rays and other waves by diffraction. It comprises a set of rings called Fresnel zones which cause the incident wave to alternately shift its phase or change its amplitude. An example of a zone plate with an opaque centre, and alternating transparent and opaque circular zones is shown below. The radius to the centre of successive zones is denoted r_m, where m is an integer. When plane waves with wavelength λ are incident normally on this zone plate, they are brought to a focus at **F** along the optical axis at distance f from the zone plate centre (see the side view profile).



(i) Consider a wave passing through the transparent circular zone with radius r_m (where m is odd), and another wave passing straight through the centre of the zone plate if it were to be transparent. Write down the optical path difference δ_m between the peripheral wave and this straight-through wave in terms of r_m and f.

If $\delta_m = \dfrac{m\lambda}{2}$, show that this produces constructive interference at ***F***, and so r_m is related to f and λ by: $r_m = \sqrt{m\lambda f + \dfrac{m^2\lambda^2}{4}}$. [10 marks]

(ii) The zone plate exhibits large chromatic aberration. If the chromatic aberration coefficient C_{ch} is defined as $C_{ch} = \dfrac{df}{d\lambda}$, derive an expression for C_{ch} of this zone plate under the condition that $\dfrac{m\lambda}{4} << f$. [10 marks]

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