NATIONAL UNIVERSITY OF SINGAPORE

PC1142 PHYSICS II

(Semester I: AY 2012–13)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains 5 short questions in Part I and 3 long questions in Part II. It comprises 9 printed pages.
2. Answer ALL questions.
3. Answers are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The total marks for Part I is 40 and that for Part II is 60.
6. You may use electronic calculators.
7. A list of physical constants and formulae is given on pp. 2 and 3.

Thermal physics and kinetic theory:

(i) Kinetic mean free path is given by $\ell = \frac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\frac{3k_B T}{m}}$.

(b) Average speed: $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$.

(c) Most-probable speed: $v_{mp} = \sqrt{\frac{2k_B T}{m}}$.

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$.

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T .$$

(v) One form of adiabat: $p \cdot V^{\gamma} = constant$.

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$.

(vii) Planck radiation equation:

$$I_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)} .$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3}\, m \cdot K}{T} .$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$.

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \frac{\Delta T}{L}$.

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$.

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$.

(ii) Carnot heat-engine efficiency $e_C = 1 - \frac{T_C}{T_H}$.

(iii) Entropy $dS = \frac{dq_{rev}}{T}$.

(iv) Enthalpy $H = E + PV$.

(v) Helmholtz free energy $F = E - TS$.

(vi) Gibbs free energy $G = H - TS$.

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$,

where $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(b) Refracting-surface equation: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$.

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \frac{d}{n}$.

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}} .$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}} .$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$.

(iv) F-number: $\frac{F}{\#}=\frac{f}{D}$.

(v) Numerical aperture: $NA = n \cdot \sin\theta$.

(vi) Snell's law: $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$.

(a) Critical angle: $\sin\theta_c = n_2 / n_1$.

(b) Brewster angle: $\tan\theta_p = n_2 / n_1$.

(vii) Wave relation: $v = f \cdot \lambda$.

(viii) Abbe number: $v = \frac{n_D - 1}{n_F - n_C}$.

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta = \frac{1.22\lambda}{a}$.

(ii) Slit: first diffraction minimum is at $\sin\theta = \frac{\lambda}{a}$.

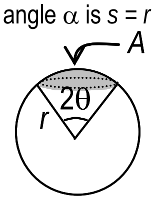
(iii) N-slit intensity pattern: $I = I_o \cdot \frac{\sin^2(N\cdot\phi/2)}{\sin^2(\phi/2)}$, where $\phi = \frac{2\pi}{\lambda}\cdot d\cdot\sin\theta$.

(iv) Single-slit diffraction pattern:

$I = I_o \cdot \frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta = \frac{2\pi}{\lambda}\cdot a \cdot \sin\theta$.

General:

(i) Geometry:
The arc length on a circle of radius r subtended by angle α is $s = r\,\alpha$.



The surface area A subtended by polar angle 2θ is:

$A = 2\,\pi r^2\,(1-\cos\theta)$.

(ii) Logarithms and exponents :

$\log_a (b\,c) = \log_a b + \log_a c$

$\log_a b = \log_d b / \log_d a$

$a^b * a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

(iii) Integrations:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ except for $n = -1$ for which $\int x^{-1} dx = \ln x + c$.

(iv) Taylor expansions:

$\sin\theta = \theta - \frac{1}{6}\theta^3 + ...$

$\cos\theta = 1 - \frac{1}{2}\theta^2 + ...$

$\tan\theta = \theta + \frac{1}{3}\theta^3 + ...$

$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + ...$

(v) Series sum:

$\sum_{n=0}^{n=m} a_0 r^n = \frac{a_o(1-r^{m+1})}{1-r}$

Universal constants:

Gas constant R = 8.314 J K^{-1} mol^{-1}

Boltzmann constant k_B = 1.381x10^{-23} J K^{-1}

Stefan–Boltzmann constant σ = 5.670x10^{-8} W m^{-2} K^{-4}

Speed of light in vacuum c = 2.998x10^8 m s^{-1}

Avogadro's number N_A = 6.022x10^{23} mol^{-1}

Permittivity of free space ε_o = 8.854x10^{-12} Fm^{-1}

Planck constant h = 6.626x10^{-34} J s

Part 1 Answer ALL questions. Each question carries 8 marks.

1. Consider a planet with radius r_P orbiting a star of radius r_S at a mean distance r_O from the centre of the star. Assume this star is a uniform blackbody with temperature T_S. Assume the planet is a rapidly rotating uniform greybody with emissivity e_P and albedo α_P. The albedo is the ratio of the average power reflected from the surface of the planet to the incident power upon it, for the emission spectrum of the star. Furthermore assume that the planet has a uniform steady-state surface temperature T_P. Derive an expression for T_P.

[8 marks]

2. Acetylene is a symmetric linear molecule with the molecular structure H−C≡C−H. Deduce the molar heat capacity at constant volume for this gas, assuming that none of its vibrational modes contribute to the heat capacity.

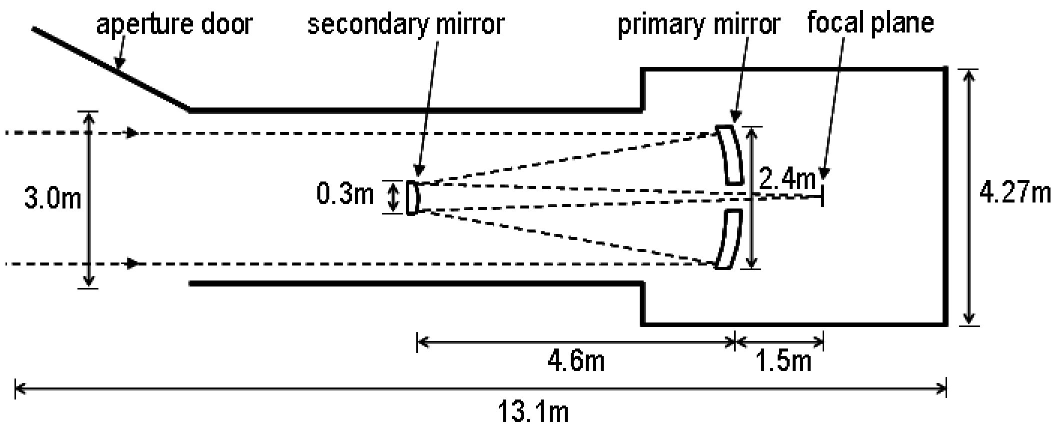
[8 marks]

3. Hydrogen gas (H_2 (g)) and oxygen gas (O_2 (g)) can be chemically combined in a fuel cell to give liquid water (H_2O (ℓ)) and produce electrical power. If this reaction takes place under the usual standard conditions of 1 atm and 298 K, compute the maximum amount of electrical work that can be obtained per mole of liquid H_2O produced. The standard enthalpy of formation $\Delta H^o_{f,298}$ and entropy S^o_{298} are given in the table below.

Quantity	H_2O (ℓ)	H_2 (g)	O_2 (g)
$\Delta H^o_{f,298}$ (kJ mol^{-1})	–285.83	0	0
S^o_{298} (J K^{-1} mol^{-1})	69.91	130.68	205.14

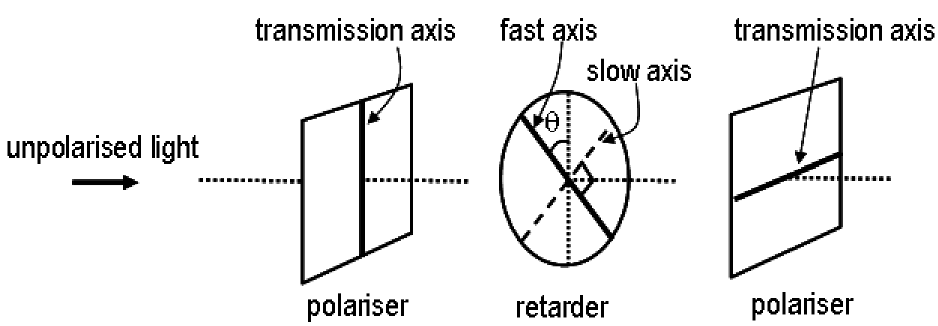
[8 marks]

4. The Hubble Space Telescope uses a 2.4-m diameter hyperbolic (concave-like) mirror to collect light. Other dimensions are as shown below. Compute its angular resolution for green light based on the Rayleigh's criteria.



[8 marks]

5. Unpolarised light of a particular wavelength and intensity is incident on a sequence of ideal polariser−wave retarder plate−polariser, as shown below. The transmission axis of the first polariser is oriented vertically, that of the second polariser horizontally, and the fast axis of the retarder is rotated at θ = 45° to the vertical, as shown below. This retarder causes light polarized parallel to the slow axis to be retarded by a phase shift of π relative to the light polarized parallel to the fast axis. Briefly explain the state of polarisation and intensity of the light that exits the second polariser.



[8 marks]

Part 2 Answer ALL questions. All questions carry 20 marks each.

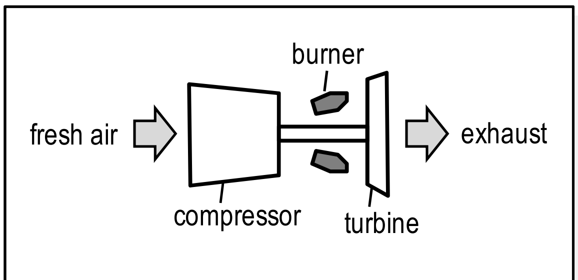
6. A gas turbine comprises a compressor, a burner and a turbine, as shown below on the left. Its operation can be modeled by the Brayton cycle with 4 steps (as shown below, right):

(a) Adiabatic compression of working gas (fresh air) by the compressor, 1 to 2;

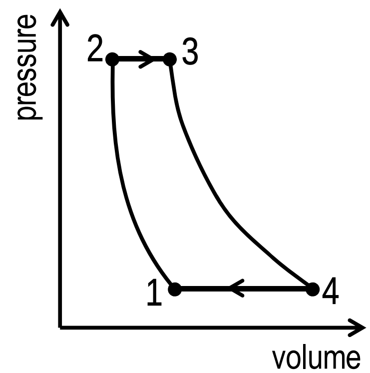
(b) Isobaric heating of the gas by the burner, 2 to 3;

(c) Adiabatic expansion of the gas at the turbine, 3 to 4;

(d) Isobaric cooling of the gas (exhaust) to the initial state, 4 to 1.



Schematic of a gas turbine



(i) Sketch the temperature *vs* entropy state diagram for this cycle, taking care to label all the points. [6 marks]

(ii) Show that the temperature ratio for the turbine and the compressor are the same, i.e., $\frac{T_3}{T_4} = \frac{T_2}{T_1}$. Hence or otherwise, show that the thermal efficiency of this cycle is given by $e_{th} = 1-(r_p)^{(1-\gamma)/\gamma}$, where γ is the heat capacity ratio and r_p is the pressure ratio given by $r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4}$. [8 marks]

(iii) For a particular gas turbine, the pressure ratio is 6.0, the inlet temperature at the compressor (T_1) is 30°C, the inlet temperature at the turbine (T_3) is 1200°C, and the flow rate of air is 0.2 kg s^{-1}. Compute the net power output of this turbine. For air, γ = 1.4 and c_p = 1.005 kJ kg^{-1} K^{-1}. [6 marks]

7. A plano-convex lens has a flat surface and a convex surface. One such lens is used to form an image of the sun on a screen. The lens has a diameter of 10 cm, and is made of glass with a refractive index of 1.55. This glass is transparent to solar radiation, which is mainly in the visible and near infrared spectral region. The radius of curvature of the convex surface is 33 cm. The sun subtends a full angle of 0.533° at the earth, and provides an intensity of 1 kW m^{-2}.

(i) Compute the required distance of the screen from the lens in the thin-lens approximation. [6 marks]

(ii) Compute the image diameter of the sun. [6 marks]

(iii) Compute the image intensity of the sun, neglecting reflections at the lens surfaces. [4 marks]

(iv) Briefly explain whether the image quality is expected to be different if the flat surface or convex surface of the lens is facing the sun, assuming perfect alignment? [4 marks]

8. A Michelson interferometer consists of two highly reflective flat mirrors M_1 and M_2, located at distances d_1 and d_2 respectively from the centre of a flat beam-splitter *C* (see Fig. 8-1 below). A point source *S* illuminates *C*, which reflects half of the incident light intensity towards M_2, as shown by solid rays in Fig. 8-2 below; and transmits the other half towards M_1, as shown by dotted rays in Fig. 8-2. M_2 reflects the light back to *C*, which transmits half of it to the observation plane *E*. Similarly M_1 reflects the light back to *C*, which reflects half of it to *E*. These two beams overlap and interfere at *E* to give a series of concentric bright fringes.

The formation of these fringes can be understood as follows (see also Fig. 8-3): Reflection of *S* in *C* and then M_2 produces a virtual source S_2'' located below M_2. Reflection of *S* in M_1 and then *C* produces another virtual source S_1''. Since S_2'' and S_1'' are both derived from *S*, the light waves emitted from them are coherent.

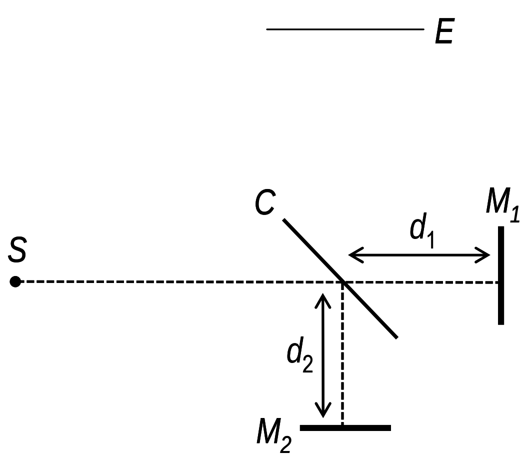


Fig. 8-1

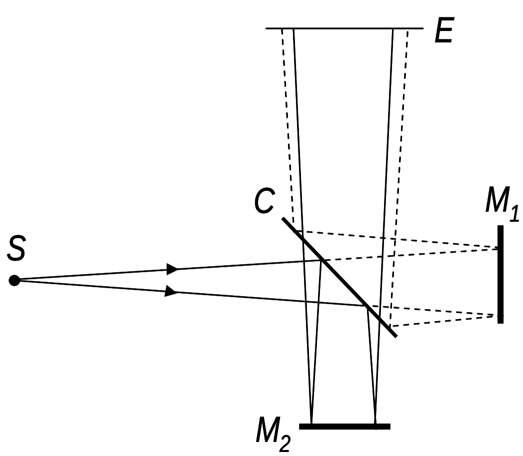


Fig. 8-2

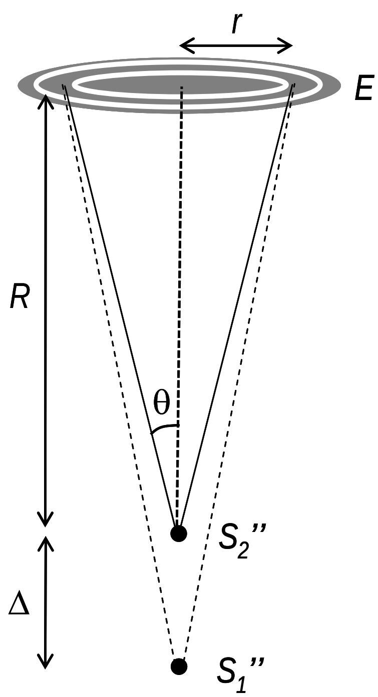


Fig. 8-3

The distance from S_2'' to E is denoted R, the radius of a bright fringe is denoted r, the ray angle from S_2'' to the bright fringe is θ, and the separation between S_2'' and S_1'' is Δ.

(i) Using the object–image relationship for mirrors, derive the relationship between Δ, d_1 and d_2. [6 marks]

(ii) For the case of $R >> \Delta$, show that the bright fringes occur at values of θ which satisfy the relationship:

$\Delta \cos \theta = m \lambda$, where m is an integer, and λ is the wavelength of light. [6 marks]

(iii) Hence or otherwise, show that the radii of the bright fringes r are given by:

$r = R\sqrt{2\left(1 - \frac{m\lambda}{\Delta}\right)}$, for $m\lambda \leq \Delta$. Explain what is observed on the observation plane E if Δ is an integer multiple of λ.

[8 marks]

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