1. (i) mass of N_2 molecule $\quad m = \dfrac{28.0\ \text{g mol}^{-1}}{6.022\times10^{23}\ \text{mol}^{-1}} = 4.65\times10^{-26}\ \text{kg}$

root-mean-square speed $\quad v_{rms} = \sqrt{\dfrac{3k_BT}{m}} = \sqrt{\dfrac{(3)(1.381\times10^{-23}\ \text{J K}^{-1})(298\ \text{K})}{(4.65\times10^{-26}\ \text{kg})}} = 515\ \text{ms}^{-1}$

(ii) number density $\quad n_v = \dfrac{p}{k_BT} = \dfrac{101325\ \text{Pa}}{(1.381\times10^{-23}\ \text{JK}^{-1})(298\ \text{K})} = 2.46\times10^{25}\ \text{m}^{-3}$

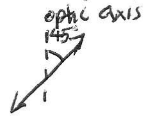
mean free path $\quad l = \dfrac{1}{\sqrt{2}\pi n_v d^2} = \dfrac{1}{\sqrt{2}\pi(2.46\times10^{25}\ \text{m}^{-3})(3\times10^{-10}\ \text{m})^2} = 100\ \text{nm}$

2. (i) phase of o-ray $\quad \phi_o = -k_o z = -\dfrac{2\pi}{\lambda_0} n_o d$

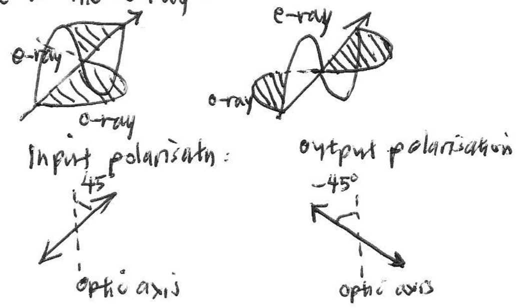
phase of e-ray $\quad \phi_e = -k_e z = -\dfrac{2\pi}{\lambda_0} n_e d$

difference $\quad \Phi = \phi_o - \phi_e = \dfrac{2\pi}{\lambda_0}(n_e - n_o)d$

(ii)



after passing half-wave retarder, the o-ray is phase-shifted by π relative to the e-ray:

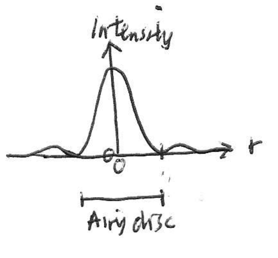


and so the output light is still linearly polarised, but rotated to $-45°$.

3. (i) Angular width of Airy disc $\quad \theta_{Airy} = \dfrac{1.22\lambda}{a} = \dfrac{1.22\ (550\times10^{-9}\ \text{m})}{2.4\ \text{m}} = 2.8\times10^{-7}\ \text{rad}$ ← green light

Diameter of Airy disc on image plane $\quad d_{Airy} = 2f\theta_{Airy} = 2(57.6\ \text{m})(2.8\times10^{-7}\ \text{rad}) = 32\ \mu\text{m}$

(ii)



4. (i) $C_2H_2(g) + 2\frac{1}{2}O_2(g) \rightarrow 2CO_2(g) + H_2O(g) \quad \Delta H^\circ_{comb,298} = ?$

$$\Delta H^\circ_{comb,298} = 2\Delta H^\circ_{f,298}[CO_2(g)] + \Delta H^\circ_{f,298}[H_2O(g)] - \Delta H^\circ_{f,298}[C_2H_2(g)] - 2\tfrac{1}{2}\Delta H^\circ_{f,298}[O_2(g)]$$

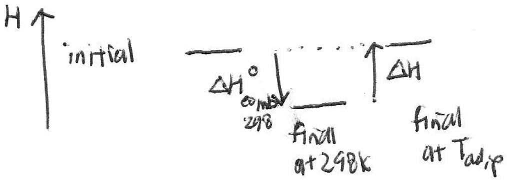
$$= 2(-393.5) + (-241.8) - (226.7) - 0 \quad kJ\,mol^{-1}$$

$$= -1255.5\ kJ\,mol^{-1}$$

(ii) Change in enthalpy of system of products with temperature

$$\Delta H = H(T) - H(298K) = \sum_{prod, i} n_i \bar{C}_{p_i} \Delta T = 2\bar{C}_p[CO_2(g)](T-298) + \bar{C}_p[H_2O(g)](T-298) + 2\tfrac{1}{2}\bar{C}_p[O_2(g)](T-298) + 20\bar{C}_p[N_2(g)](T-298)$$

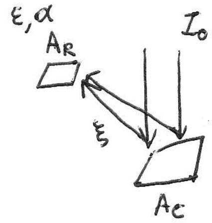
$T_{ad,p}$ is the temperature where $\Delta H^\circ_{comb,298} + \Delta H = 0$



$$\text{so} \quad \underbrace{-1{,}255.5}_{\Delta H^\circ_{comb,298}} + \Big[2\underbrace{(54.3\times10^{-3})}_{CO_2} + \underbrace{(41.2\times10^{-3})}_{H_2O} + 2\tfrac{1}{2}\underbrace{(34.9\times10^{-3})}_{O_2} + 20\underbrace{(32.7\times10^{-3})}_{N_2}\Big](T_{ad,p} - 298) = 0$$

$$T_{ad,p} = 1700K$$

5 (i)



Power on collection mirror	$I_o \cdot A_c$
Reflected power on receiver	$\xi \cdot I_o \cdot A_c$
Absorbed power by receiver	$\alpha \cdot \xi \cdot I_o \cdot A_c$
Radiated power by receiver	$\sigma A_R \varepsilon T_R^4$ (Stefan - Boltzman law)

(i) Heat input to engine $Q_H = \alpha\xi I_o A_c - \sigma A_R \varepsilon T_R^4$

(ii) Efficiency of absorption

$$\eta_R = \frac{\alpha\xi I_o A_c - \sigma A_R \varepsilon T_R^4}{I_o A_c}$$

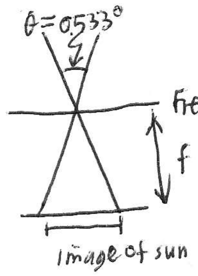
$$= \alpha\xi - \frac{\sigma A_R T_R^4}{I_o A_c}$$

Carnot efficiency $\eta_c = \left(1 - \frac{T_o}{T_R}\right)$

Net efficiency $\eta = \eta_R \cdot \eta_c = \left(\alpha\xi - \frac{\sigma A_R}{I_o A_c} T_R^4\right)\left(1 - \frac{T_o}{T_R}\right)$

6.

(i)



Diameter of sun image $= f\theta = (1.5\text{m})\left(\frac{0.533^\circ}{360^\circ}\times 2\pi\right)$

$= 1.4\text{ cm}$

Size of solar cell required is a square with side 1.4 cm, minimum.

(ii) Image intensity $I_{image} = I_0 \cdot \frac{A_{lens}}{A_{image}} = (1.0\text{ kW m}^{-2})\left(\frac{30\times 30\text{cm}^2}{\pi(0.70\text{cm})^2}\right)$

$= 585\text{ kW m}^{-2}$

(iii) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

R_2: ∞, because plano-convex

$f = 1.5\text{m}$

$n = 1.60$

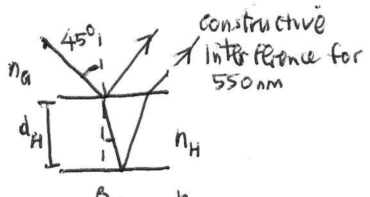
$R_1 = 0.90\text{ m}$

(iv) Image quality is poor, due to scattering at the rings.

7

(i) The periodicity of the stack causes constructive interference in reflection for a fundamental wavelength of 550 nm. Near infrared radiation (700 – 2000nm) does not experience the constructive interference in reflection and hence passes through the stack.

(ii)



Snell's law, across multilayers

$n_i \sin\theta_i = n_H \sin\theta_{\beta_H}$

$1.00 \sin 45^\circ = 2.20 \sin\theta_{\beta_H}$

$\theta_{\beta_H} = 18.7^\circ$

Thin-film interference, for constructive reflection:

$$\underbrace{-\frac{2\pi}{\lambda_0} 2 n_H d_H \cos\beta_H}_{\text{optical path}} + \underbrace{(0 - \pi)}_{\text{reflection phase shifts}} = 2m\pi \qquad m \in \mathbb{Z}$$

We need m to be the smallest admissible so that the fundamental wavelength is 550nm,

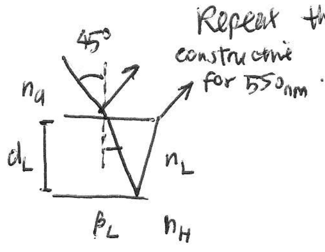
hence $m = -1$

So $\quad -\frac{2\pi}{\lambda_0} 2n_H d_H \cos\beta - \pi = -2\pi$

for $\lambda_0 = 550\,\text{nm}$
$n_H = 2.20$
$\beta_H = 18.7°$

we get $\quad d_H = 66\,\text{nm}$

Repeat the same for d_L :



$$n_a \sin\theta_i = n_L \sin\beta_L$$
$$1.00 \sin 45° = 1.35 \sin\beta_L$$
$$\theta_{\beta L} = 31.6°$$

For constructive interference:

$$-\frac{2\pi}{\lambda_0} 2n_L d_L \cos\beta_L + (\pi - 0) = 2m\pi \qquad m \in \mathbb{Z}$$

Now $m = 0$

so $\quad -\frac{2\pi}{\lambda_0} 2n_L d_L \cos\beta_L + \pi = 0$

for $\lambda_0 = 550\,\text{nm}$
$n_L = 1.35$
$\beta_L = 31.6°$
we get $\ d_L = 120\,\text{nm}$

(iii) The centre wavelength obeys the equation:

$$-\frac{2\pi}{\lambda_{ctr}} 2n_H d_H \cos\beta_H - \pi = -2\pi \qquad \text{(for ZnS layer)} \quad \text{-- Eq(1)}$$

where $\cos\beta = (1 - \sin^2\beta_H)^{1/2}$

$$\sin^2\beta_H = \left(\frac{n_a}{n_H}\right)^2 \sin^2\theta_i$$

so $\quad \cos\beta = \left[1 - \left(\frac{n_a}{n_H}\right)^2 \sin^2\theta_i\right]^{1/2}$

Eq (1) becomes

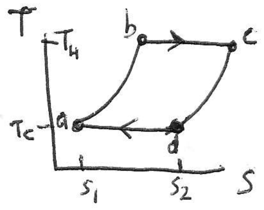
$$\lambda_{ctr} = 4n_H d_H \left[1 - \left(\frac{n_a}{n_H}\right)^2 \sin^2\theta_i\right]^{1/2} = \lambda_{ctr}$$

Differentiate wrt θ_i

$$\frac{d\lambda_{ctr}}{d\theta_i} = -\frac{4n_H d_H}{\left[1 - \left(\frac{n_a}{n_H}\right)^2 \sin^2\theta_i\right]^{1/2}} \cdot \left(\frac{n_a}{n_H}\right)^2 \sin\theta_i \cos\theta_i$$

8

(i)



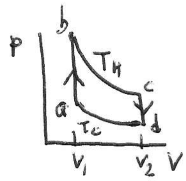
$a \to b$: $\Delta S = \int_{T_C}^{T} \frac{dq_{rev}}{T} = \int_{T_C}^{T} \frac{n C_v dT}{T} = n C_v \ln\left(\frac{T}{T_C}\right)$

$b \to c$: T is constant, S increases: $\Delta S = R \ln\left(\frac{V}{V_1}\right)$

$c \to d$: $\Delta S = \int_{T_H}^{T} \frac{dq_{rev}}{T} = \int_{T_H}^{T} \frac{-n C_v dT}{T} = -n C_v \ln\left(\frac{T}{T_H}\right)$

$d \to a$: $\Delta S = R \ln\left(\frac{V}{V_2}\right)$

(ii) $e_{th} = \frac{W_{out}}{Q_{in}} = \frac{\oint T dS}{\int^{c} T dS} = \frac{\frac{T_H - T_C}{S_2 - S_1}}{\frac{T_H}{S_2 - S_1}} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$



$$e_{th} = \frac{W_{out}}{Q_{in}}$$

$$W_{out} = \int_b^c P dV + \int_d^a P dV$$

$$= \int_{V_1}^{V_2} nRT_H \frac{dV}{V} + \int_{V_2}^{V_1} nRT_C \frac{dV}{V}$$

$$= nR \ln\left(\frac{V_2}{V_1}\right)[T_H - T_C]$$

$$Q_{in} = \int_a^b dq$$

$$= nR \ln\left(\frac{V_2}{V_1}\right)(T_H)$$

$$e_{th} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$$

(iii) The efficiency of this idealized Stirling cycle is identical to that of the Carnot cycle. This is because heat input occurs isothermally at T_H, and heat output occurs isothermally at T_C.

Isothermal processes are not practical for engine cycles because it occurs too slowly.