NATIONAL UNIVERSITY OF SINGAPORE

PC1142 PHYSICS II

(Semester I: AY 2013–14)

Examiner: Assoc Prof Peter Ho

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your matriculation number in the answer booklet. DO NOT WRITE YOUR NAME.
2. This examination paper contains 5 short questions in Part I and 3 long questions in Part II. It comprises 9 printed pages.
3. Answer ALL questions.
4. This is a CLOSED BOOK examination.
5. You may use electronic calculators to make your answers.
6. A list of physical constants and formulae is given on pp. 2 and 3.

Thermal physics and kinetic theory:

(i) Kinetic mean free path is given by $\ell = \frac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\frac{3k_BT}{m}}$.

(b) Average speed: $v_{av} = \sqrt{\frac{8k_BT}{\pi m}}$.

(c) Most-probable speed: $v_{mp} = \sqrt{\frac{2k_BT}{m}}$.

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$.

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T.$$

(v) One form of adiabat: $p \cdot V^\gamma = constant$.

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$.

(vii) Planck radiation equation:

$$I_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)}.$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3}\, m \cdot K}{T}.$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$.

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \frac{\Delta T}{L}$.

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$.

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$.

(ii) Carnot heat-engine efficiency $e_C = 1 - \frac{T_C}{T_H}$.

(iii) Entropy $dS = \frac{dq_{rev}}{T}$.

(iv) Enthalpy $H = E + PV$.

(v) Helmholtz free energy $F = E – TS$.

(vi) Gibbs free energy $G = H – TS$.

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, where $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(b) Refracting-surface equation: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$.

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \frac{d}{n}$.

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}}.$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}}.$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$.

(iv) *F*-number: $\frac{F}{\#}=\frac{f}{D}$.

(v) Numerical aperture: $NA = n\cdot\sin\theta$.

(vi) Snell's law: $n_1\cdot\sin\theta_1 = n_2\cdot\sin\theta_2$.

(a) Critical angle: $\sin\theta_c = n_2 / n_1$.

(b) Brewster angle: $\tan\theta_p = n_2 / n_1$.

(vii) Wave relation: $v = f\cdot\lambda$.

(viii) Abbe number: $v=\frac{n_D-1}{n_F-n_C}$.

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta=\frac{1.22\lambda}{a}$.

(ii) Slit: first diffraction minimum is at $\sin\theta=\frac{\lambda}{a}$.

(iii) *N*-slit intensity pattern: $I=I_o\cdot\frac{\sin^2(N\cdot\phi/2)}{\sin^2(\phi/2)}$, where $\phi=\frac{2\pi}{\lambda}\cdot d\cdot\sin\theta$.

(iv) Single-slit diffraction pattern:

$I=I_o\cdot\frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta=\frac{2\pi}{\lambda}\cdot a\cdot\sin\theta$.

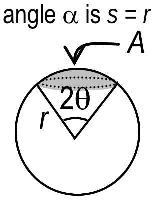
(v) Thin-film interference:

$\delta = 2\cdot n\cdot d\cdot\cos\beta$, where β is the refracted angle.

General:

(i) Geometry:

The arc length on a circle of radius *r* subtended by angle α is $s = r\,\alpha$.



The surface area *A* subtended by polar angle 2θ is:

$A = 2\,\pi r^2\,(1-\cos\theta)$.

(ii) Logarithms and exponents :

$\log_a(b\,c) = \log_a b + \log_a c$

$\log_a b = \log_d b / \log_d a$

$a^b * a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

(iii) Integrations:

$\int x^n dx=\frac{x^{n+1}}{n+1}+c$, except for $n = -1$ for which $\int x^{-1}dx=\ln x + c$.

(iv) Taylor expansions:

$\sin\theta=\theta-\frac{1}{6}\theta^3+\ldots$

$\cos\theta=1-\frac{1}{2}\theta^2+\ldots$

$\tan\theta=\theta+\frac{1}{3}\theta^3+\ldots$

$(1+x)^a=1+ax+\frac{a(a-1)}{2}x^2+\ldots$

(v) Series sum:

$\sum_{n=0}^{n=m} a_0 r^n=\frac{a_o(1-r^{m+1})}{1-r}$

(vi) Differentiations:

$d(\sin u) = \cos u\,du$

$d(\cos u) = -\sin u\,du$

$d(u^m) = m u^{m-1}\,du$

Universal constants:

Gas constant R = 8.314 J K^{-1} mol^{-1}

Boltzmann constant k_B = 1.381x10^{-23} J K^{-1}

Stefan–Boltzmann constant σ = 5.670x10^{-8} W m^{-2} K^{-4}

Speed of light in vacuum c = 2.998x10^8 m s^{-1}

Avogadro's number N_A = 6.022x10^{23} mol^{-1}

Permittivity of free space ε_o = 8.854x10^{-12} Fm^{-1}

Planck constant h = 6.626x10^{-34} J s

Part 1. Answer ALL questions. Each question carries 8 marks.

1. Assume that air can be modeled by molecules with a molecular diameter of 3.0 Å, and molar mass of 28.0 g mol^{-1}. The temperature of this air is 298 K, and its pressure is 101325 Pa. [NB: 1 Å = 10^{-10} m]

   (i) Compute the root-mean-square speed of an air molecule.

   (ii) Compute the mean free path of an air molecule.

[8 marks]

2. Retarders change the state of polarisation of light. A retarder can be made from a plate of a birefringent material in which the *o*-ray (i.e., ray with linear polarisation perpendicular to optic axis) and the *e*-ray (i.e., polarisation parallel to optic axis) travel with different velocities. If these rays are initially in phase, they develop a phase difference Φ after passing the retarder.

   (i) Show that Φ of the *o*-ray relative to the *e*-ray is given by $\Phi = \frac{2\pi}{\lambda_o}(n_e - n_o)d$, where n_e is the refractive index for the *e*-ray, n_o is the refractive index for the *o*-ray, and *d* is the thickness of the plate.

   (ii) If the retarder is a half-wave plate, i.e., $\Phi = \pi$, and the incident light is linearly polarised at an angle of 45º relative to the optic axis of the retarder, explain the state of polarisation of this light after passing the retarder.

[8 marks]

3. The Hubble Space Telescope (HST) has an aperture diameter of 2.4-m set by its primary mirror, and a focal length of 57.6 m.

   (i) Compute the size of the Airy disk, i.e., radius of the first dark ring, at the image plane for green light.

   (ii) Sketch the image intensity profile for a star and mark on this plot the edge of the Airy disc. [NB: Image intensity profile is the plot of intensity *vs* distance from the centre of the image.]

[8 marks]

4. The constant-pressure adiabatic flame temperature $T_{ad,p}$ is the temperature reached by the combustion products when the fuel is burnt at constant pressure without any heat loss to the surroundings and without any non-PV work done. This means the enthalpy of a closed system of fuel, oxidant and any other substances present in the mixture in its initial state at 298 K equals the enthalpy of the system of combustion products and other substances in the final state at $T_{ad,p}$. The combustion of acetylene (C_2H_2) in 200% theoretical air is given by:

$$C_2H_2(g) + 2.5\ O_2(g) + 2.5\ O_2(g) + 20\ N_2(g) \rightarrow 2CO_2(g) + H_2O(g) + 2.5\ O_2(g) + 20\ N_2(g)$$

The table below gives some thermochemical data. $\Delta H^{o}_{f,,298}$ is the enthalpy of formation of the substance at standard pressure and 298 K, S^{o}_{298} is the entropy at standard pressure and 298 K, c_p is the molar heat capacity at constant pressure, and c_v is the molar heat capacity at constant volume. Both c_p and c_v are given for 1000 K, and may be taken to be the average heat capacities over the temperature range of interest.

	$\Delta H^{o}_{f,,298}$ (kJ mol^{-1})	S^{o}_{298} (J K^{-1} mol^{-1})	c_p (J K^{-1} mol^{-1})	c_v (J K^{-1} mol^{-1})
C_2H_2 (g)	226.7	200.8	44.1	35.8
O_2 (g)	0	205.2	34.9	26.6
N_2 (g)	0	192.0	32.7	24.4
CO_2 (g)	–393.5	213.8	54.3	45.8
H_2O (g)	–241.8	188.8	41.2	32.8

~~[8 marks]~~

(i) Compute the enthalpy of combustion at 298 K and standard pressure.

(ii) Write down an expression for the change in enthalpy of the system of substances shown on the right hand side of the equation corresponding to a temperature change from 298 K to T. Hence or otherwise, compute $T_{ad,p}$ for the combustion of acetylene in 200% theoretical air.

[8 marks]

5. A concentrated solar power (CSP) system uses mirrors or lenses to concentrate solar power onto a heat receiver. This heat receiver drives a heat engine, often a steam turbine. One such system has a mirror concentrator with a total collection area A_C, and collection efficiency ξ which gives the ratio of reflected power on the receiver to the incident power on the collection mirror. The heat receiver has an area A_R, absorptivity α for solar radiation, and emissivity ε for thermal radiation. During operation, it reaches a steady-state temperature T_R, set by the condition that heat gain from the concentrator equals the sum of heat input to the engine Q_H and heat loss by thermal radiation Q_{rad} at the receiver. A_R is also the effective area for this thermal radiation. The heat engine operates between T_R and a low temperature bath T_O that corresponds to the environment. The incident solar intensity is I_O.

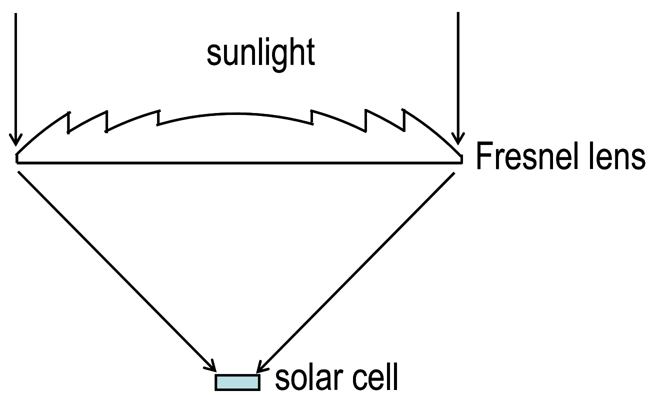
(i) Write down an expression for Q_H in terms of I_O, A_C, A_R, T_R and any other necessary parameters.

(ii) Hence or otherwise, write down an expression for the maximum solar-to-electricity energy conversion efficiency η of this CSP in terms of T_O, T_R and any other necessary parameters. You may wish to start from $\eta = \eta_R \cdot \eta_{Carnot}$, where η_R is the overall efficiency for transfer of incident solar power to the heat engine, and η_{Carnot} is the Carnot efficiency of the engine.

[8 marks]

Part 2. Answer ALL questions. All questions carry 20 marks each.

6. A concentrated photovoltaic (CPV) system uses mirrors or lenses to focus sunlight onto a small but highly efficient and expensive solar cell to generate electricity. In one design, a lightweight and inexpensive plastic plano-convex Fresnel lens is used to focus sunlight onto the solar cell, as shown below. The plastic material is polycarbonate which has a refractive index of 1.60. The Fresnel lens has a square shape with side length of 30 cm and a focal length of 1.5 m. The solar cell also has a square shape. The sun subtends a full angle of 0.533° at the earth, and provides an intensity of 1.0 kW m^{-2} at lens surface.



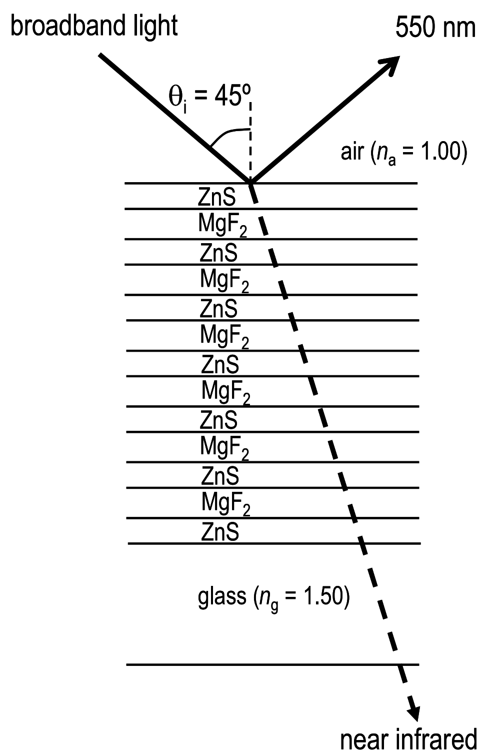
(i) Compute the minimum size of the solar cell required to capture all the sunlight at the image plane of this CPV design. [5 marks]

(ii) Compute the intensity of sunlight at the solar cell, neglecting reflections at the lens surface. [5 marks]

(iii) Compute the required radius of curvature of the curved surface of this Fresnel lens in first order paraxial ray theory. [5 marks]

(iv) Briefly explain whether Fresnel lenses would be useful to make high-quality images for photography. [5 marks]

7. A dielectric mirror makes use of thin-film interference to reflect light. A cold mirror is an example of a dielectric mirror that reflects visible light but transmits near infrared radiation. A simple cold mirror fabricated using a single periodic stack of transparent high-index ZnS (refractive index n_H = 2.20) and transparent low-index MgF_2 (n_L = 1.35) is shown below. It is designed to strongly reflect light with a centre-wavelength λ_{ctr} of 550 nm, when this light falls on the mirror at an angle of incidence θ_i of 45º.



(i) Compute the required thicknesses of the ZnS and MgF_2 layers. [8 marks]

(ii) Consider thin-film interference inside a single ZnS layer in the middle of the stack. Show that λ_{ctr} varies with θ_i according to:

$$\frac{d\lambda_{ctr}}{d\theta_i} = -4n_H d_H \left(\frac{n_a}{n_H}\right)^2 \frac{\sin\theta_i \cos\theta_i}{\left(1-\left(\frac{n_a}{n_H}\right)^2 \sin^2\theta_i\right)^{1/2}}.$$ [8 marks]

(iii) Briefly explain why this periodic stack is able to strongly reflect visible light but not near infrared wavelengths. [4 marks]

8. A concentrated power system can also employ a Stirling engine to convert the input heat to work. A Stirling engine is a closed cycle engine that contains a fixed mass of working gas that is cycled between a hot heat exchanger (such as the solar heat receiver) and a cold heat exchanger (usually the environment). An idealised Stirling cycle is shown below. This can be used as a starting point to analyse the behaviour of a practical Stirling engine, even though it is a poor representation of what actually goes on in the engine. The idealised Stirling cycle comprises the following steps:

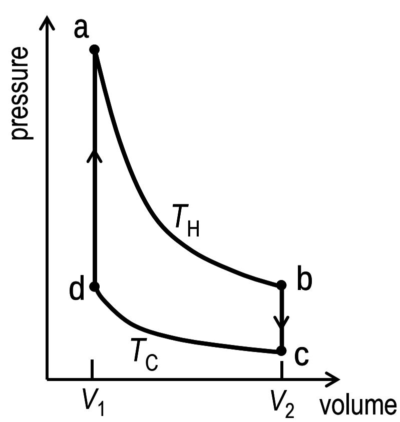
a→b: isothermal expansion of a hot gas with heat input from the hot heat exchanger;

b→c: isochoric cooling of the hot gas by heat loss to an internal heat regenerator;

c→d: isothermal compression of the cold gas with heat output to the cold heat exchanger;

d→a: isochoric heating of the cold gas by heat gain from the internal heat regenerator.

External heat input into the cycle occurs only over a→b, while external heat output from the cycle occurs only over c→d. The regenerator performs an important role to internally store heat energy lost by the gas over b→c and returns this to the gas over d→a.



(i) Sketch the temperature *vs* entropy state-variable diagram for this cycle. Indicate on the diagram the locations of a, b, c and d, and the direction of the cycle. [8 marks]

(ii) Hence or otherwise, derive an expression for the thermal efficiency of this cycle in terms of T_H, T_C and any other necessary parameters. [8 marks]

(iii) Briefly comment on the result from (ii), and comment also on whether the isothermal processes are practical for engine cycles. [4 marks]

--END OF PAPER ---

PH