1. (i) mean free path $\ell = \dfrac{1}{\sqrt{2}\pi N_d d^2} = \dfrac{1}{\sqrt{2}\pi (10^6 cm^{-3})(0.29 \times 10^{-7} cm)^2} = 2.7 \times 10^8$ cm

(ii) mean speed $\quad V_{av} = \left(\dfrac{8 k_B T}{\pi m}\right)^{1/2} = \left(\dfrac{8(1.38 \times 10^{-23} J K^{-1})(20K)}{(2.0 \times 10^{-3} kg\,mol^{-1} / 6.022 \times 10^{23} mol^{-1})}\right)^{1/2} = 460 m s^{-1}$

mean time between collision $\quad t = \dfrac{\ell}{V_{av}} = \dfrac{2.7 \times 10^6 m}{460 ms^{-1}} = 5900 s \approx 1.6h$

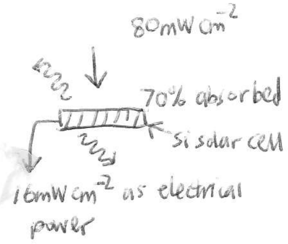
2. (i) The energy equipartition theorem states that in thermal equilibrium, all degrees-of-freedom that give quadratic energy dependence have an average energy of $\frac{1}{2}k_B T$ each.

(ii) The trapped particle has quadratic energy dependence of its potential energy on distance, since $u = \frac{1}{2}\alpha r^2$. Hence for each degree of freedom, x, y and z in r,

$$\langle u \rangle = \frac{1}{2} k_B T$$

$$\frac{1}{2}\alpha \langle x^2 \rangle = \frac{k_B T}{\alpha}$$

$$x_{rms} = \sqrt{\frac{k_B T}{\alpha}}$$



3. Intensity absorbed by cell, $I_{abs} = (80 mW cm^{-2})(0.70) = 56 mW cm^{-2}$

Heat to be dissipated $\quad I_{ex} = 56 mW cm^{-2} - 16 mW cm^{-2} = 40 mW cm^{-2}$

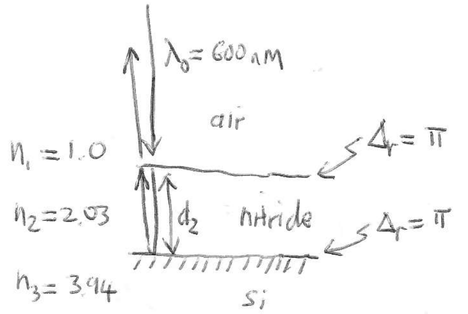
Assuming radiation is main heat loss mechanism, $I_{rad} = I_{ex}$

Since the solar cell has two faces (top and bottom),

$$I_{rad} = 2\underbrace{\sigma}_{(5.67 \times 10^{-8})(10^{-4}) W cm^{-2} K^{-4}} \overbrace{e}^{0.96}(T^4 - \overbrace{T_{surr}}^{298K}{}^4) = 40 mW cm^{-2}$$

This gives $\quad T = 327$ K.

4 (i)



Since both $\Delta_r = \pi$, destructive interference occurs if the round-trip optical path in the nitride layer is a multiple of $\frac{\lambda_0}{2}$:

$$\delta_2 = 2 n_2 d_2 = m \frac{\lambda_0}{2} \qquad m \in \mathbb{Z}^+$$

The thinnest d_2 is given by $m = 1$,

$$d_2 = \frac{\lambda_0}{4 n_2} = \frac{600nm}{4(2.03)} = 74nm$$

(ii) [Since $n_2 = \sqrt{n_1 n_3}$, complete destructive interference occurs for $\lambda_0 = 600nm$.]

The optical path difference between the first and second reflected rays $\delta_2 = 300\,nm$.

Complete destructive interference occurs

for $m \frac{\lambda_0}{2} = \delta_2$, where $m \in$ odd integers

which gives

m	λ_0
1	600 nm
2	200 nm

reflectivity minima

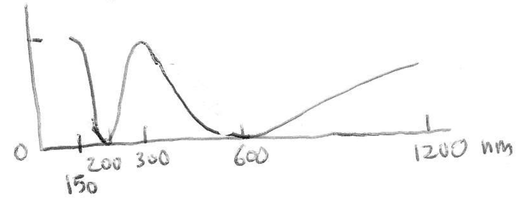
Constructive interference occurs

for $m \lambda_0 = \delta_2$ where $m \in \mathbb{Z}^+$

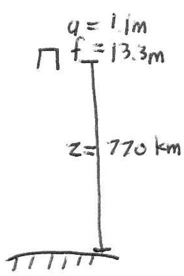
which gives

m	λ_0
1	300 nm
2	150 nm

reflectivity maxima

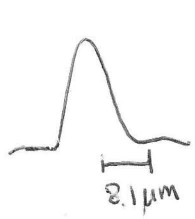


5.



Diffraction-limited resolution for panchromatic imaging

$$\theta_{res} = \frac{1.22\lambda}{a} = \frac{1.22(550\times10^{-9}m)}{1.1m} = 0.61\,\mu rad$$



The required pixel size on the focal plane is

$$d_{im} = f\,\theta_{res} = (13.3m)(0.61\,\mu rad) = 8.1\,\mu m$$

which is available. Therefore resolution is not limited by pixel size.

(i) Diffraction-limited resolution, on the ground, for panchromatic imaging,

$$d_{ground} = z\,\theta_{res} = (770\times10^3\,m)(0.61\,\mu rad) = 0.47m$$

(ii) Since $d_{ground} = z\left(\frac{1.22\lambda}{a}\right)$, to improve resolution significantly, by a factor of 5 or so, $\frac{a}{z}$ needs to be increased by that factor.
Hence orbital height needs to be decreased by a factor of 5; or aperture diameter needs to be increased by a factor of 5 together with shrinking pixel size by a factor of 5, or some combination of the two.

6. (i) It means that the optical system forms an image of an object an infinite distance away, typically more than 100f away. Parallel rays are brought to focus on the focal plane

(ii) In the paraxial ray approximation, for a spherical refracting surface,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Consider the cornea surface, for object at infinity $p = \infty$,

$$\frac{1.34}{q_a} = \frac{1.34 - 1.00}{7.70\,mm}$$

which gives $q_a = 30.3\,mm$

Consider the first lens face, $p_b = 3.6 - 30.3 = -26.7mm$

$$\frac{1.34}{-26.7mm} + \frac{1.41}{q_b} = \frac{1.41 - 1.34}{10.0mm}$$

which gives $q_b = 24.7\,mm$

Consider the second lens face $p_c = 3.6 - 24.7 = -21.1mm$

$$\frac{1.41}{-21.1mm} + \frac{1.34}{q_c} = \frac{1.34 - 1.41}{-6.00\,mm}$$

which gives $q_c = 17.1\,mm$

Hence the thickness of the vitreous humor should be 17.1 mm on the optical axis

(iii) To accomodate a near object, the effective focal length of the lens system needs to be shorter, ie, refractive power needs to be higher. The lens has higher index than the anterior chamber and vitreous humor so to achieve higher refractive power, its radii of curvature need to become smaller.

7 (i) $\frac{Q_E}{Q_G}$ gives the ratio of heat pumped in the cycle to the energy input (as heat) to the cycle. One often has to pay for this heat input, or specially arrange for it. The desired outcome is the pumping of the heat.

(ii) First law of thermodynamics: for one complete cycle, net heat input and net work input sums to zero. Thus

$$Q_E + Q_G - Q_C - Q_A = 0 \quad \ldots Eq(1)$$

Second law of thermodynamics: for one complete cycle $\Delta S_{surr} \geq 0$. Thus

$$-\frac{Q_E}{T_E} - \frac{Q_G}{T_G} + \frac{Q_C}{T_0} + \frac{Q_A}{T_0} \geq 0 \quad \ldots Eq(2)$$

From Eq(1), Eq(2) becomes

$$-\frac{Q_E}{T_E} - \frac{Q_G}{T_G} + \frac{Q_E + Q_G}{T_0} \geq 0$$

Now divide throughout by Q_G,

$$\frac{Q_E}{Q_G}\left(-\frac{1}{T_E}\right) - \frac{1}{T_G} + \frac{Q_E}{Q_G}\left(\frac{1}{T_0}\right) + \frac{1}{T_0} \geq 0$$

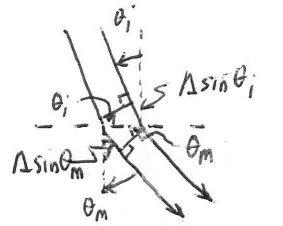
$$\frac{Q_E}{Q_G} \leq \left(\frac{1}{T_G} - \frac{1}{T_0}\right)\left(\frac{1}{T_0} - \frac{1}{T_E}\right)^{-1}$$

$$\leq \left(1 - \frac{T_0}{T_G}\right)\frac{T_E}{T_0 - T_E}$$

(iii) Decrease T_0 towards T_E, increase T_G well above T_0.

8 (i) $\Lambda \sin\theta_m = m\lambda_0$

(ii)



Optical path difference of ray ② relative to ray ①.

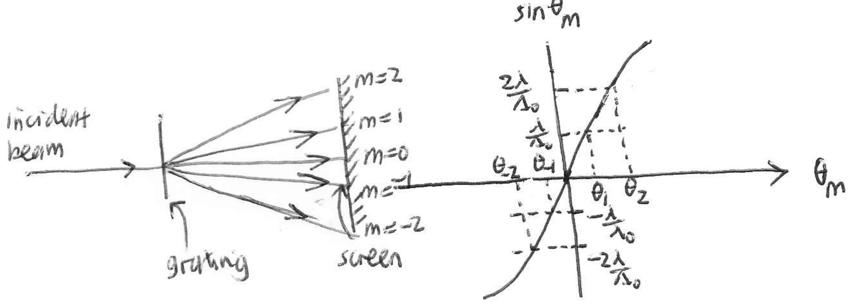
$$\Lambda\sin\theta_m - \Lambda\sin\theta_i$$

For constructive interference, this equals to $m\lambda_0$. Thus

$$\Lambda(\sin\theta_m - \sin\theta_i) = m\lambda_0$$

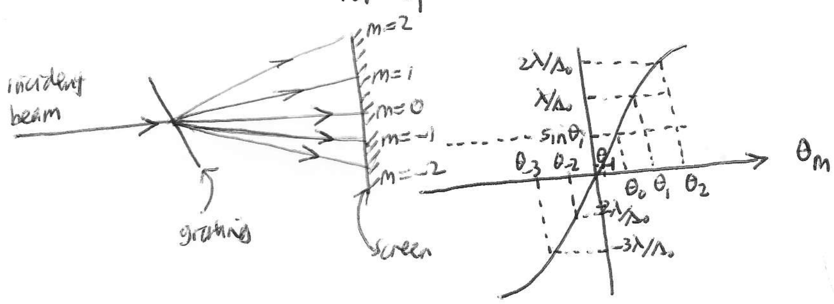
Let us examine the solution

For $\theta_i = 0$



The $m=0$ mode is the straight through mode. The positive and negative modes are symmetrically placed on each side of the $m=0$ mode.

For $\theta_i > 0$



The $m=0$ mode is still the straight through mode. The positive and negative modes are no longer symmetrically placed on each side of the $m=0$ mode.

(iii) Longer wavelengths need a larger diffraction angle to accumulate the required phase difference.