NATIONAL UNIVERSITY OF SINGAPORE

PC1142 PHYSICS II

(Semester I: AY 2014–15)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your matriculation number in the answer booklet. Do not write your name.
2. This examination paper comprises 8 printed pages with 5 short questions in Part I and 3 long questions in Part II.
3. Answer all questions.
4. This is a closed book examination.
5. You may use electronic calculators.
6. A list of physical constants and formulae is given on pp. 2 and 3.

[Current version has been updated with typographical corrections.]

Thermal physics and kinetic theory of gases:

(i) Kinetic mean free path is given by $\ell = \frac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\frac{3k_B T}{m}}$.

(b) Average speed: $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$.

(c) Most-probable speed: $v_{mp} = \sqrt{\frac{2k_B T}{m}}$.

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$.

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T \cdot$$

(v) One form of adiabat: $p \cdot V^{\gamma} = constant$.

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$.

(vii) Planck radiation equation:

$$I_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)} \cdot$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3}\, m \cdot K}{T} \cdot$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$.

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \frac{\Delta T}{L}$.

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$.

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$.

(ii) Carnot heat-engine efficiency $e_C = 1 - \frac{T_C}{T_H}$.

(iii) Entropy $dS = \frac{dq_{rev}}{T}$.

(iv) Enthalpy $H = E + PV$.

(v) Helmholtz free energy $F = E - TS$.

(vi) Gibbs free energy $G = H - TS$.

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$,

where $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(b) Refracting-surface equation:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}.$$

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \frac{d}{n}$.

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}}.$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}}.$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$.

(iv) F-number: $\frac{F}{\#}=\frac{f}{D}$.

(v) Numerical aperture: $NA = n \cdot \sin\theta$.

(vi) Snell's law: $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$.

(a) Critical angle: $\sin\theta_c = n_2 / n_1$.

(b) Brewster angle: $\tan\theta_p = n_2 / n_1$.

(vii) Wave relation: $v = f \cdot \lambda$.

(viii) Abbe number: $v = \frac{n_D - 1}{n_F - n_C}$.

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta = \frac{1.22\lambda}{a}$.

(ii) Slit: first diffraction minimum is at $\sin\theta = \frac{\lambda}{a}$.

(iii) N-slit intensity pattern: $I = I_o \cdot \frac{\sin^2(N\cdot\phi/2)}{\sin^2(\phi/2)}$,

where $\phi = \frac{2\pi}{\lambda}\cdot d \cdot \sin\theta$.

(iv) Single-slit diffraction pattern:

$I = I_o \cdot \frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta = \frac{2\pi}{\lambda}\cdot a \cdot \sin\theta$.

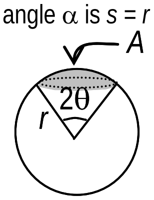
(v) Thin-film interference:

$\delta = 2 \cdot n \cdot d \cdot \cos\beta$, where β is the refracted angle.

General:

(i) Geometry:

The arc length on a circle of radius r subtended by angle α is $s = r\,\alpha$.



The surface area A subtended by polar angle 2θ is:

$A = 2\,\pi r^2\,(1 - \cos\theta)$.

(ii) Logarithms and exponents :

$\log_a (b\,c) = \log_a b + \log_a c$

$\log_a b = \log_d b / \log_d a$

$a^b * a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

(iii) Integrations:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$,

except for $n = -1$, where $\int x^{-1} dx = \ln x + c$.

(iv) Taylor expansions:

$\sin\theta = \theta - \frac{1}{6}\theta^3 + ...$

$\cos\theta = 1 - \frac{1}{2}\theta^2 + ...$

$\tan\theta = \theta + \frac{1}{3}\theta^3 + ...$

$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + ...$

(v) Series sum:

$\sum_{n=0}^{n=m} a_0 r^n = \frac{a_o(1-r^{m+1})}{1-r}$

(vi) Differentiations:

$d(\sin u) = \cos u\, du$

$d(\cos u) = -\sin u\, du$

$d(u^m) = m u^{m-1}\, du$

Universal constants:

Gas constant R = 8.314 J K^{-1} mol^{-1}

Boltzmann constant k_B = 1.381x10^{-23} J K^{-1}

Stefan–Boltzmann constant σ = 5.670x10^{-8} W m^{-2} K^{-4}

Speed of light in vacuum c = 2.998x10^8 m s^{-1}

Avogadro's number N_A = 6.022x10^{23} mol^{-1}

Permittivity of free space ε_o = 8.854x10^{-12} Fm^{-1}

Planck constant h = 6.626x10^{-34} J s

Part 1. Answer ALL questions. Each question carries 8 marks.

1. The interstellar medium is thought to have a cold dense phase consisting of clouds of neutral hydrogen molecules (H_2) at a temperature of about 20 K and a density of 10^6 molecules cm^{-3}. The molecular diameter of H_2 is 0.29 nm.

   (i) Compute the mean free path between collisions of these H_2 molecules.

   (ii) Hence or otherwise, compute the mean time between their collisions.

[8 marks]

2. An optical trap is a device that uses a highly focused light beam to hold onto and move microscopic or nanometer-sized dielectric particles. The trap provides a spherical potential that has quadratic energy dependence on displacement, i.e., the potential energy of the particle is given by $U = \frac{1}{2}\alpha r^2$, where α is the "stiffness" of the trap assumed to be independent of *r*, and *r* is the displacement of the particle from centre of the trap.

   (i) Write down a statement of the energy equipartition theorem.

   (ii) Using this theorem, derive an expression for the root-mean-square displacement $< x^2 >^{1/2}$ of the trapped particle from the trap centre in thermal equilibrium at temperature *T*.

[8 marks]

3. A large thin solar cell is mounted horizontally by a frame which provides negligible heat conduction. The temperature of the surroundings is 298 K. This cell is illuminated by sunlight at normal incidence with an intensity of 80 mW cm^{-2}. The cell absorbs 70% of this intensity and produces electrical power corresponding to 16 mW cm^{-2}. The rest of the absorbed intensity is dissipated as heat. The net heat loss by thermal radiation to the surroundings at temperature T_{surr} is given by $I_{rad} = \sigma e(T^4 - T_{surr}^4)$, where σ is the Stefan–Boltzmann constant, *e* is the emissivity of the surface and *T* is the surface temperature. For the solar cell, *e* = 0.96. Assuming that radiation is the main heat loss mechanism, compute the steady-state temperature of the cell.

[8 marks]

4. An anti-reflection coating made of silicon nitride is to be deposited onto the surface of a silicon solar cell to minimize its surface reflectivity in air at normal incidence and 600-nm wavelength. Silicon and silicon nitride have refractive indices of 3.94 and 2.03 respectively at this wavelength.

(i) Compute the smallest film thickness of the silicon nitride that can meet this requirement.

(ii) Briefly explain whether you expect any other reflectivity minima and any reflectivity maxima to occur between 150 nm and 1200 nm, assuming the refractive indices are constant.

[8 marks]

5. GeoEye-1 is a high-resolution earth observation satellite launched in 2008. It orbits the earth at an altitude of 770 km. It carries a telescope that has an aperture diameter of 1.1 m, defined by its primary concave mirror, and a focal length of 13.3 m implemented using folded optics. Images are captured on a pixelated array located at the focal plane. The pixels used for panchromatic (i.e., black-and-white) imaging has a size of 8.1 μm. The centre wavelength for panchromatic imaging is 550 nm.

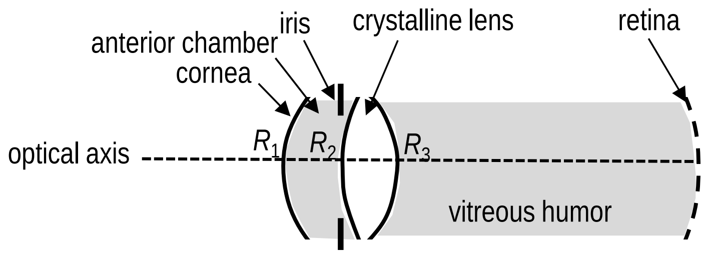
(i) Assuming that the optics is diffraction-limited, compute the resolution that the satellite can provide for panchromatic imaging.

(ii) Briefly explain what changes are needed to improve the panchromatic resolution significantly, such as to 0.1 m.

[8 marks]

Part 2. Answer ALL questions. All questions carry 20 marks each.

6. The Gullstrand's three-surface schematic model of the human eye is shown below.



In this model, the cornea, assumed to be negligibly thin, defines the first surface of the anterior chamber. Light passes through the anterior chamber, the crystalline lens and the vitreous humor before it falls on the retina. For the relaxed eye focused at infinity, the cornea has a radius of curvature of 7.70 mm (R_1), the crystalline lens has radii of curvature of 10.0 mm (R_2) and –6.00 mm (R_3), and the image plane lies on the retina. The refractive indices for 587-nm light (n_{587}) and thicknesses (t) measured along the optical axis are shown below.

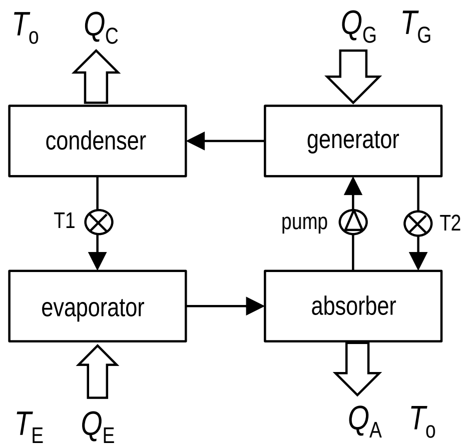
	n_{587}	t (mm)
anterior chamber	1.34	3.60
crystalline lens	1.41	3.60
vitreous humor	1.34	?

(i) Briefly explain the meaning of "focused at infinity". [4 marks]

(ii) Compute the required thickness of the vitreous humor in the paraxial ray approximation. [12 marks]

(iii) The radii of curvature of the lens are controlled by the ciliary muscles. Briefly explain whether these radii of curvature should become larger or smaller for the eye to accommodate a near object. [4 marks]

7. An absorption refrigerator is a heat pump that uses heat instead of work as the input energy. An absorption refrigeration cycle is shown below. The vapour refrigerant at high pressure is condensed to the liquid state in the condenser. This releases heat Q_C (enthalpy of condensation) to the surroundings at constant ambient temperature T_0. The liquid refrigerant is then admitted into the evaporator at low pressure by throttle valve T1. The refrigerant evaporates and absorbs heat Q_E at constant temperature T_E, where $T_E < T_0$. The vapour refrigerant then flows into the absorber where it is absorbed by a liquid absorbent. This releases Q_A (enthalpy of absorption) to the surroundings at T_0. The absorbent mixture is pumped to the generator which receives heat input at a constant temperature T_G, where $T_G > T_0$. The mixture receives heat Q_G to regenerate the vapour refrigerant at T_G and high pressure. This vapour flows to the condenser and completes the cycle. A second throttle valve T2 allows the absorbent to return to the absorber. A small amount of mechanical work is required to operate the pump, but this is negligible because the liquid absorbent mixture is incompressible.



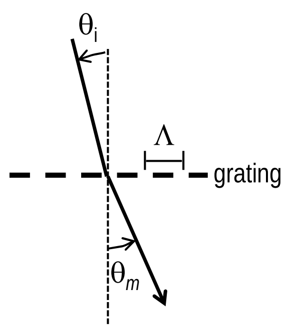
(i) The coefficient of performance of this absorption refrigerator is defined by $\mathrm{COP} = \dfrac{Q_E}{Q_G}$. Briefly explain why this definition is meaningful. [4 marks]

(ii) Using the first law of thermodynamics, write down an expression relating Q_E, Q_A, Q_C and Q_G. Hence using the second law of thermodynamics (hint: the entropy statement) or otherwise, show that the maximum COP is given by $\dfrac{T_E}{T_o - T_E}\left(1 - \dfrac{T_o}{T_G}\right)$. [12 marks]

[*question to be continued on next page*]

(iii) Briefly explain how to improve the COP for a desired T_E through control of T_o and T_G. [4 marks]

8. A beam of lightwaves with planar wavefronts and vacuum wavelength λ_o is incident normally (i.e., $\theta_i = 0$) on a transmission diffraction grating in air as shown below. The grating has a periodicity of Λ. The grating splits the beam into several beams traveling at different θ_m, where m is the mode number.



(i) For $\theta_i = 0$, write down an expression that relates θ_m to λ_o, Λ and any other necessary parameters. [4 marks]

(ii) If the incident beam is tilted by θ_i, derive an expression that relates θ_m to θ_i, and any other necessary parameters. Hence or otherwise, briefly explain how the diffraction pattern changes on the observation screen in the far field as the grating is tilted, assuming a few m modes are visible on the screen. [12 marks]

(iii) Briefly explain why a longer wavelength exhibits a larger θ_m than a short wavelength for the same m. [4 marks]

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