1. Power conducted per unit area $P_{cond} = -k\frac{dT}{dr} = -(3.0\,W m^{-1} K^{-1})(-20\times10^{-3}\,K m^{-1}) = 60\,mW\,m^{-2}$

Area of earth's surface $A = 4\pi r^2 = 4\pi(6371\times10^3 m)^2 = 5.10\times10^{14}\,m^2$

Global geothermal power is $(60\,mW\,m^{-2})(5.10\times10^{14}\,m^2) = 3\times10^{13}\,W = 30\,TW$

2. (i) The energy equipartition theorem states that in thermal equilibrium, all degrees-of-freedom that give quadratic energy dependence have an average energy of $\frac{1}{2}k_BT$ each.

(ii) In 2D

$$\langle \tfrac{1}{2}mv_{2D}^2\rangle = \tfrac{2}{2}k_BT$$

$$v_{2D,rms} = \sqrt{\frac{2k_BT}{m}} = \left(\frac{2(1.38\times10^{-23}\,JK^{-1})(298K)}{\left(\frac{720.6\times10^{-3}\,kg\,mol^{-1}}{6.022\times10^{23}\,mol^{-1}}\right)}\right)^{1/2} = 83\,ms^{-1}$$

3. Fuel-cell reaction: $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$ $\quad\Delta G^\circ_{rxn}, \Delta H^\circ_{rxn}$ at 298K, 1.00 atm

(i) Max available electrical energy is given by $-\Delta G^\circ_{rxn,298}$

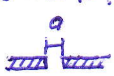
$$\begin{aligned}\Delta G^\circ_{rxn,298} &= \Delta G^\circ_{f,298}(H_2O(l)) - \Delta G^\circ_{f,298}(H_2(g)) - \tfrac{1}{2}\Delta G^\circ_{f,298}(O_2(g))\\ &= -306.66 - (-38.94) - \tfrac{1}{2}(-61.13)\ \ kJ\,mol^{-1}\\ &= -237.16\ kJ\,mol^{-1}\end{aligned}$$

Heat energy is given by $-\Delta H^\circ_{rxn,298}$

$$\begin{aligned}\Delta H^\circ_{rxn,298} &= \Delta H^\circ_{f,298}(H_2O(l)) - \Delta H^\circ_{f,298}(H_2(g)) - \tfrac{1}{2}\Delta H^\circ_{f,298}(O_2(g))\\ &= -285.83 - 0 - \tfrac{1}{2}(0)\ \ kJ\,mol^{-1}\\ &= -285.83\ kJ\,mol^{-1}\end{aligned}$$

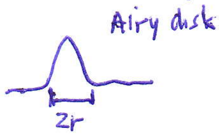
Max thermodynamic efficiency $= \dfrac{-\Delta G^\circ_{rxn,298}}{-\Delta H^\circ_{rxn,298}} = \dfrac{237.16}{285.83} = 83\%$

(ii) The fuel cell reaction does not involve converting heat to work.



4. (i) Diameter of Airy disk on focal plane

$$2r = 2f\theta = 2\times1.22\lambda\frac{f}{a} = 2\times1.22\lambda(f/\#)$$
$$= 2\times1.22(550\,nm)(16)$$
$$= 21.5\ \mu m$$



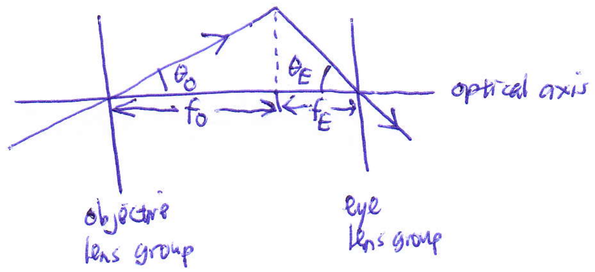
(ii) Since the diameter of the Airy disk is 5x as large as the pixel size, the image is limited by diffraction effects and not pixel-size effects. Decreasing the pixel size further will not improve image sharpness.

5. (i) For reflected beam to be completely s-polarised, the incidence angle needs to be the Brewster angle.

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.55}{1.00}\right) = 57.2°$$

(ii) Since a portion of the s-polarised light waves has been reflected, the transmitted beam is elliptically polarised with dominant p-component.

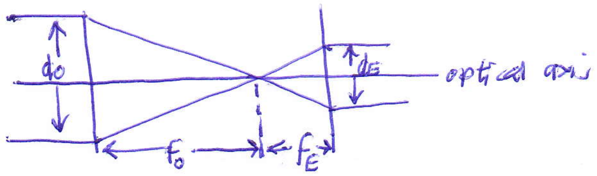
6. (i)



angular magnification $m = \frac{\theta_E}{\theta_o} = \frac{f_E}{f_o}$

The 8x binoculars make objects appear as if they are only one-eighth of the distance away. The angular separation between object points is 8X as large, allowing one to resolve details better..

(ii)



$$\frac{d_E}{d_o} = \frac{f_E}{f_o}$$

(iii)
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_{OR}}$$
p: 3.0m f_{OR}: 0.135m

$$q = 0.141 \text{ m}$$

The image of the object at ∞ is formed on the focal plane. The image of the object at 3.0m is formed 6 mm beyond the focal plane. The eye lens group has to be translated 6 mm away from the objective lens group to keep the image at infinity.

(iv) The achromat corrects for chromatic aberration. Chromatic aberration occurs as a result of optical dispersion which causes different wavelength to have different focal length: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

7. (i) Thermal efficiency, $e_{th} = 1 - \frac{Q_C}{Q_H}$

where

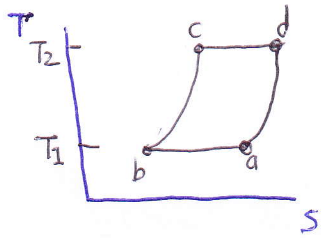
$$Q_C = C_p(T_2 - T_1) + RT_1 \ln\left(\frac{V_a}{V_b}\right) = C_p(T_2 - T_1) + RT_1 \ln\left(\frac{P_2}{P_1}\right)$$

$$Q_H = C_p(T_2 - T_1) + RT_2 \ln\left(\frac{V_d}{V_c}\right) = C_p(T_2 - T_1) + RT_2 \ln\left(\frac{P_2}{P_1}\right)$$

(ii) For perfect regeneration, heat input to cycle occurs only at $c \to d$, while heat output occurs at $a \to b$. Heat output at $d \to a$ exactly balanc heat input at $b \to c$.

Hence thermal efficiency, $e_{th} = 1 - \frac{RT_1 \ln\left(\frac{P_b}{P_a}\right)}{RT_2 \ln\left(\frac{P_c}{P_d}\right)} = 1 - \frac{T_1}{T_2}$

(iii)



$$\Delta S_{a \to b} = R \ln\left(\frac{V_b}{V_a}\right) = R \ln\left(\frac{P_1}{P_2}\right) \quad \text{per mol of gas}$$

$$\Delta S_{b \to c} = C_p \ln\left(\frac{T_2}{T_1}\right) \quad \text{per mol of gas}$$

$$\Delta S_{c \to d} = R \ln\left(\frac{V_d}{V_a}\right) = R \ln\left(\frac{P_2}{P_1}\right)$$

$$\Delta S_{d \to a} = C_p \ln\left(\frac{T_1}{T_2}\right)$$

(iv) Gas temperature rises due to adiabatic heating, $T \propto V^{1-\gamma}$, where γ is the heat capacity ratio $\frac{C_p}{C_v}$.

8. (i) 4. There are two secondary maxima between two adjacent primary maxima. Number of secondary maxima equals $N-2$, where N is the number of slits illuminated.

(ii) The angular half-width of the diffraction envelope given by $\sin\theta_d = \frac{\lambda_0}{a}$ is $\approx 5.5°$.

Hence $a = \frac{\lambda_0}{\sin\theta_d} = \frac{488\text{nm}}{\sin(5.5°)} = 5.1\ \mu m$.

(iii) The angular separation between adjacent primary maxima (m=0 and m=1) given by $\Lambda \sin\theta_{int} = \lambda_0$ is $\approx 1.4°$.

Hence $\Lambda = \frac{\lambda_0}{\sin\theta_{int}} = \frac{488\text{nm}}{\sin(1.4°)} = 20\ \mu m$.

(iv) The angular half-width of the diffraction envelope would be given by $\sin\theta_d = \frac{\lambda_0}{na}$, and the angular separation between adjacent primary maxima (m=0 and m=1) would be given by $\Lambda \sin\theta_{int} = \frac{\lambda_0}{n}$. In first order approximation, the diffraction pattern would be compressed by factor n.