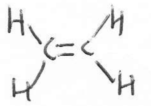
1. 

(i) Molecule has three translational degrees of freedom and three rotational degrees of freedom. Each degree of freedom contributes $\frac{1}{2} k_B T$ to the average internal energy of the molecule, according to the energy equipartition theorem.

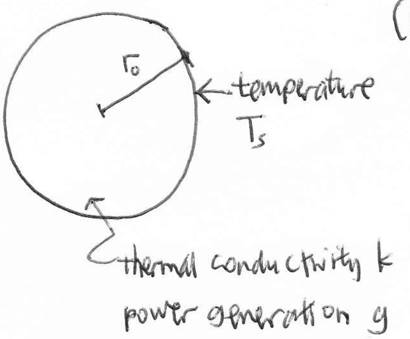
Hence $C_V = 3R$ where R is the . . gas constant.

(ii) $C_P = C_V + R$ for ideal gas,

Hence $C_P = 4R$ for ethylene

and $\gamma = \frac{C_P}{C_V} = \frac{4}{3}$

2.



(i) At steady state, heat produced by sphere must equal heat convected at its surface.

Hence

$$\frac{4}{3}\pi r_0^3 g = h\,(T_s - T_0) \cdot 4\pi r_0^2$$

which gives

$$T_s = T_0 + \frac{g r_0}{3h}$$

(ii) Define T_c to be temperature at the centre of the sphere.

Consider infinitesimal thin spherical shells with width dr. Heat flow is radial.

The 1D heat conduction equation gives

$$\frac{dQ}{dt} = -k \cdot 4\pi r^2 \cdot \frac{dT}{dr}$$

but $\frac{dQ}{dt} = \frac{4}{3}\pi r^3 g$

Hence $\frac{4}{3}\pi r^3 g = -k\, 4\pi r^2 \frac{dT}{dr}$

$$\frac{gr}{3k} = -\frac{dT}{dr}$$

Integrating both sides with relevant boundary conditions

$$\int_0^{r_0} \frac{gr}{3k}\,dr = \int_{T_c}^{T_s} -dT$$

$$\frac{g r_0^2}{6k} = T_c - T_s$$

Hence $$T_c = T_s + \frac{g r_0^2}{6k}$$

3. (i) Maximum amt of electrical work that can be obtained is given by $-\Delta G_{rxn}$, where

$$\Delta G_{rxn} = \Delta H_{rxn} - T\Delta S_{rxn}$$

For the MeOH fuel cell rxn, all at standard conditions

$$\Delta H_{rxn} = \Delta H_f^\circ(CO_2) + 2*\Delta H_f^\circ(H_2O(l)) - \Delta H_f^\circ(CH_3OH(l)) - 1.5*\Delta H_f^\circ(O_2)$$

$$= -393.51 + 2(-\quad) - (-238.4) - 0 \quad kJ\,mol^{-1}$$

$$= -726.77\ kJ\,mol^{-1}$$

$$\Delta S_{rxn} = S^\circ(CO_2) + 2*S^\circ(H_2O(l)) - S^\circ(CH_3OH(l)) - 1.5*S^\circ(O_2)$$

$$= 213.79 + 2(69.91) - 127.2 - 1.5*(205.14) \quad JK^{-1}mol^{-1}$$

$$= -81.3\ JK^{-1}mol^{-1}$$

$$\Delta G_{rxn} = -726.77 - (298)(-81.3\times10^{-3}) \quad kJ\,mol^{-1}$$

$$= -702.5\ kJ\,mol^{-1}$$

(ii) The amt of electrical work

because the entropy also decreases, and so the 2nd Law requires a minimum amt of heat to be lost to the environment to compensate for this entropy reduction.

4. (i) Change in specific entropy

$$\Delta s = \int_{T_1}^{T_2} \frac{dQ_{rev}}{T} = \int_{T_1}^{T_2} \frac{C_p}{M_r} \cdot \frac{dT}{T} = \frac{C_p}{M_r} \ln \frac{T_2}{T_1}$$

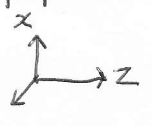
For air, $C_p = \frac{7}{2} R$

Hence $\Delta s = \frac{\frac{7}{2}R}{M_r} \ln \frac{T_2}{T_1} = \frac{29.1\ \mathrm{J K^{-1} mol^{-1}}}{28.8 \times 10^{-3}\ \mathrm{kg\ mol^{-1}}} \ln \left(\frac{598\mathrm{K}}{298\mathrm{K}} \right)$

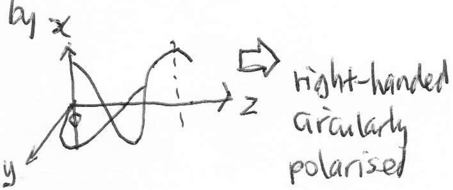
$= 704\ \mathrm{J K^{-1} kg^{-1}}$

(ii) No. Entropy is a state function.

5. (i) Circularly-polarised light, right handed is given by

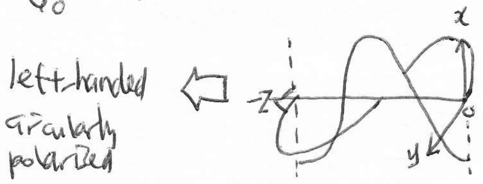


$$\xi_x = \xi_0 \cos(\omega t - kz)$$
$$\xi_y = -\xi_0 \sin(\omega t - kz)$$



Upon reflection, assuming reflection phase shift is π,

$$\xi_x = \xi_0 \cos(\omega t + kz + \pi) = -\xi_0 \cos(\omega t + kz)$$
$$\xi_y = -\xi_0 \sin(\omega t + kz + \pi) = +\xi_0 \sin(\omega t + kz)$$



the light becomes left-handed circularly polarised.

(ii) The left-handed circularly polarised light when transmitted through the same quarterwave retarder will become plane polarised but at 90° orientation with respect to the transmission axis.

As a result, it gets blocked.

Anti-glare application.

6. (i)

Heat input rate,

$$Q_{b\to c} = (h_c - h_b)\cdot\frac{dm}{dt}$$

Heat output rate,

$$-Q_{d\to a} = (h_d - h_a)\cdot\frac{dm}{dt}$$

Thermal efficiency, $e = 1 - \dfrac{h_d - h_a}{h_c - h_d}$

(ii) The expansion of the vapor from pressure P_2 to pressure P_1 ocours rapidly in the turbine without time for heat flow.

(iii) Carnot efficiency $e_c = 1 - \dfrac{T_a}{T_c}$ lowest T reached in cycle; highest T reached in cycle

7. (i) Image distance for object 5.0m away,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

For $p = 5.0\,m$, $f = 4.15\times10^{-3}\,m$, $q = 4.153$ mm

For $p = 1.0\,m$, $f = 4.15\times10^{-3}\,m$, $q = 4.167$ mm

Change in image distance is $\Delta q = 14\ \mu m$.

(ii) Magnification, lateral, $M = -\dfrac{q}{p} = \dfrac{4.167\,mm}{1.0\,m} = 4.167\times10^{-3}$

Size of image is thus $4.167\,mm \times 2.917\,mm$

This is smaller than the size of the sensor array. Hence the full writings can be captured.

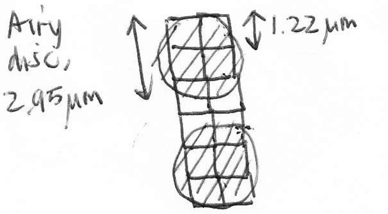
(iii) Diffraction limited spot size, angular half width

$$\sin\theta = \frac{1.22\lambda}{a}$$

Diameter $2f\tan\theta \approx 2^{*}1.22\lambda\dfrac{f}{a} \approx 2^{*}1.22\lambda(f) = 2^{*}1.22(550\times10^{-9}m)(2.2)$ ← f number

$= 2.95\ \mu m$

(iv) The pixel elements are about half the size of the Airy disc.



If a gap between two adjacent ink strokes is to be resolved, there needs to be a gap between the edges of the Airy discs. Assume a gap of 1 pixel, the centres of the Airy discs have to be separated by about 2.5 + 1.0 pixels = 3.5 pixels.

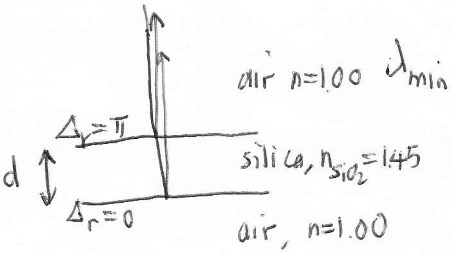
The distance on the image plane is thus

$3.5 \times 1.22\,\mu m = 4.27\,\mu m$

The corresponding distance on the object plane is $\dfrac{4.27\,\mu m}{M} = 1.0\text{ mm}$

(v) Name and briefly explain any of the achromatic aberrations.

8. (i) Transmittance valleys occur when constructive interference takes place on reflection, which causes the reflected power to be higher, and hence transmitted power lower, than neighbouring wavelengths.



Constructive interference on reflection,

$$-2\pi\frac{2nd}{\lambda_{min}} + (0-\pi) = 2m\pi$$

$$\lambda_{min} = -\frac{4nd}{2m+1}$$

where $m = -1, -2, \ldots$

(ii) The longest λ_{min} is 5800 nm. This must correspond to $m = -1$.

Hence $d = -\dfrac{\lambda_{min}(2m+1)}{4n} = -\dfrac{(5800\text{ nm})(-1)}{4(1.45)} = 1000\text{ nm}$

(iii) Perfect transmission is possible when reflection becomes zero due to completely destructive interference. This occurs when $-2\pi \frac{2nd}{\lambda_{max}} + (0 - \pi) = m\pi$

Hence all power is transmitted through the film.

(iv) If the silica film is thinned down, the interference fringes shift to shorter wavelengths. Amplitude of oscillation remains unchanged.