NATIONAL UNIVERSITY OF SINGAPORE

PC1142 Introduction to Thermodynamics and Optics

(Semester I: AY 2016–17)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your matriculation number in the answer booklet. Do not write your name.
2. This examination paper comprises 9 printed pages with 5 short questions in Part I and 3 long questions in Part II.
3. Answer all questions.
4. This is a closed book examination.
5. You may use electronic calculators.
6. A list of physical constants and formulae is given on pages 2 and 3.

Thermal physics and kinetic theory of gases:

(i) Kinetic mean free path is given by $\ell = \frac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\frac{3k_B T}{m}}$.

(b) Average speed: $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$.

(c) Most-probable speed: $v_{mp} = \sqrt{\frac{2k_B T}{m}}$.

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$.

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T \cdot$$

(v) One form of adiabat: $p \cdot V^{\gamma} = constant$.

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$.

(vii) Planck radiation equation:

$$I_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)} \cdot$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3}\, m \cdot K}{T} \cdot$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$.

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \frac{\Delta T}{L}$.

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$.

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$.

(ii) Carnot heat-engine efficiency $e_C = 1 - \frac{T_C}{T_H}$.

(iii) Entropy $dS = \frac{dq_{rev}}{T}$.

(iv) Enthalpy $H = E + PV$.

(v) Helmholtz free energy $F = E - TS$.

(vi) Gibbs free energy $G = H - TS$.

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$,

where $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(b) Refracting-surface equation:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}.$$

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \frac{d}{n}$.

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}}.$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}}.$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$.

(iv) f-number, $f/\# = \frac{f}{D}$.

(v) Numerical aperture: $NA = n \cdot \sin\theta$.

(vi) Snell's law: $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$.

(a) Critical angle: $\sin\theta_c = n_2 / n_1$.

(b) Brewster angle: $\tan\theta_p = n_2 / n_1$.

(vii) Wave relation: $v = f \cdot \lambda$.

(viii) Abbe number: $V = \frac{n_D - 1}{n_F - n_C}$.

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta = \frac{1.22\lambda}{a}$.

(ii) Slit: first diffraction minimum is at $\sin\theta = \frac{\lambda}{a}$.

(iii) N-slit intensity pattern: $I = I_o \cdot \frac{\sin^2(N \cdot \phi/2)}{\sin^2(\phi/2)}$, where $\phi = \frac{2\pi}{\lambda} \cdot d \cdot \sin\theta \cdot$

(iv) Single-slit diffraction pattern:

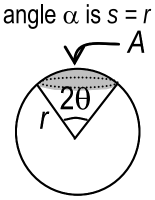
$I = I_o \cdot \frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta \cdot$

(v) Thin-film interference:

$\delta = 2 \cdot n \cdot d \cdot \cos\beta$, where β is the refracted angle.

General:

(i) Geometry:
The arc length on a circle of radius r subtended by angle α is $s = r\,\alpha$.



The surface area A subtended by polar angle 2θ is:

$A = 2\,\pi r^2\,(1 - \cos\theta)$.

(ii) Logarithms and exponents :

$\log_a (b\,c) = \log_a b + \log_a c$

$\log_a b = \log_d b / \log_d a$

$a^b * a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

(iii) Integrations:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$,

except for $n = -1$, where $\int x^{-1} dx = \ln x + c$.

(iv) Taylor expansions:

$\sin\theta = \theta - \frac{1}{6}\theta^3 + \dots$

$\cos\theta = 1 - \frac{1}{2}\theta^2 + \dots$

$\tan\theta = \theta + \frac{1}{3}\theta^3 + \dots$

$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \dots$

(v) Series sum:

$\sum_{n=0}^{n=m} a_0 r^n = \frac{a_o(1-r^{m+1})}{1-r}$

(vi) Differentiations:

d($\sin u$) = $\cos u$ du

d($\cos u$) = $-\sin u$ du

d(u^m) = $m u^{m-1}$ du

Universal constants:

Gas constant R = 8.314 J K^{-1} mol^{-1}

Boltzmann constant $k_B = 1.381 \times 10^{-23}$ J K^{-1}

Stefan–Boltzmann constant $\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}

Speed of light in vacuum $c = 2.998 \times 10^{8}$ m s^{-1}

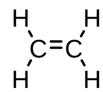
Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Permittivity of free space $\varepsilon_o = 8.854 \times 10^{-12}$ Fm^{-1}

Planck constant $h = 6.626 \times 10^{-34}$ J s

Part 1. Answer ALL questions. Each question carries 8 marks.

1. At room temperature, ethylene (CH_2CH_2) is a gas with a planar structure as shown below.



Assume ideal gas behavior and that none of its vibrational modes contributes to the heat capacity.

(i) Briefly explain and write down the expression for its molar heat capacity at constant volume (c_v) at room temperature, in terms of the gas constant R.

(ii) Hence or otherwise, write down the expression for its heat capacity ratio ($\gamma = c_p/c_v$) at room temperature.

[8 marks]

2. Consider a solid sphere of uniform material with radius r_o and constant thermal conductivity k, immersed in a fluid with temperature T_o. Heat is generated uniformly inside this sphere, at a constant rate of g in units of power per unit volume. Assume that Newton's law of cooling for heat flow: $\frac{dQ}{dt} = hA(T_s - T_o)$ can sufficiently describe the convection heat flow across its surface, where h is a temperature-independent convection coefficient, A is the surface area, T_s is the surface temperature.

(i) Show that the steady-state surface temperature of this sphere is $T_s = T_o + \frac{gr_o}{3h}$.

(ii) Hence or otherwise, show that the temperature at the centre of the sphere is $T_s + \frac{gr_o^{\,2}}{6k}$.

[8 marks]

3. Methanol fuel cells combine methanol CH_3OH (ℓ) with oxygen O_2 (g) to give water H_2O (ℓ) and carbon dioxide CO_2 (g) according to the chemical equation below to produce electrical power.

$$CH_3OH\ (\ell) + 1.5\ O_2\ (g) \rightarrow CO_2\ (g) + 2\ H_2O\ (\ell)$$

The standard enthalpy of formation $\Delta H^o_{f,298}$ and entropy S^o_{298} of these substances are given in the table below.

Quantity	CH_3OH (ℓ)	O_2 (g)	CO_2 (g)	H_2O (ℓ)
$\Delta H^o_{f,298}$ (kJ mol^{-1})	–238.4	0	–393.51	–285.83
S^o_{298} (J K^{-1} mol^{-1})	127.2	205.14	213.79	69.91

(i) Compute the maximum amount of electrical work that can be obtained per mole of CH_3OH consumed under standard conditions of 298 K and 1 atmosphere.

(ii) Briefly explain why this is less than the heat released by the same reaction when CH_3OH is directly burned with O_2.

[8 marks]

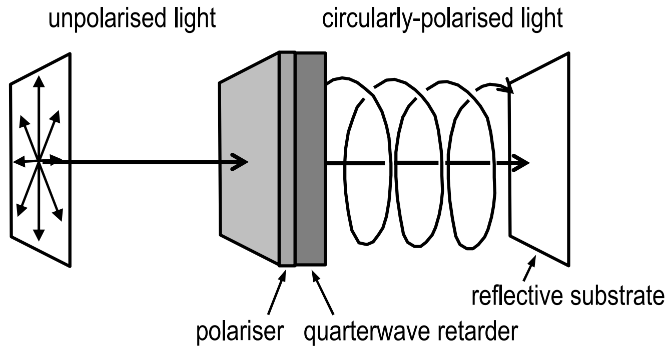
4. Air at 1.00 x 10^5 Pa and 25°C is heated reversibly at constant pressure to 325°C. Assume that air is made up of 80% N_2 and 20% O_2, with an average molecular weight M_r of 28.8 g mol^{-1} and molar heat capacity at constant pressure c_p of 3.5 R.

(i) Compute the change in the specific entropy of air. *Hint*: Specific entropy is entropy per unit mass.

(ii) Briefly explain whether this result would change if the air were heated irreversibly.

[8 marks]

5. A linear polariser film is attached to a quarterwave retarder film to make right-handed circularly polarised light as shown below. This composite film is pasted onto a highly reflective substrate. Unpolarised light is incident nearly normally on the stack.



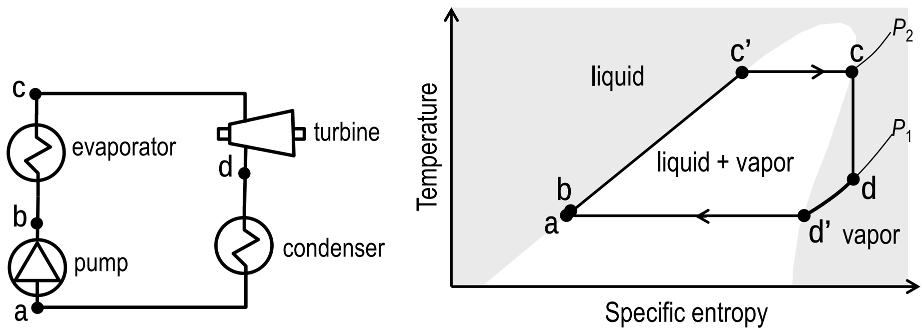
(i) Briefly explain the state of polarisation of the right-handed circularly polarised light upon reflection from the substrate.

(ii) Briefly explain whether this reflected light can be transmitted through the stack.

[8 marks]

Part 2. Answer ALL questions. Each question carries 20 marks.

6. Organic Rankine cycles (ORCs) can generate power from natural and waste heat sources, using an organic substance as their working fluid. A schematic model of an ORC is shown below together with its temperature–entropy (*TS*) diagram.



a→b	adiabatic pumping of the liquid fluid to pressure P_2 at a rate of $\frac{dm}{dt}$;
b→c'→c	isobaric heating of the liquid to the fully vapor state at pressure P_2;
c→d	adiabatic expansion of the vapor in the turbine to a mixture of liquid + vapor at pressure P_1; and
d→d'→a	isobaric condensation to the fully liquid state at pressure P_1

The points a and b lie very close to each other on the *TS* diagram. Define h_i, where i = a, b, etc, to be the specific enthalpy of the working fluid at points a, b, etc. Define s_i, where i = a, b, etc, to be the specific entropy of the working fluid at the corresponding point.

(i) Write down an expression for the rate of heat input into the working fluid during b→c'→c, in terms of the specific enthalpies h_b etc, and any other relevant parameter(s). Do the same for the rate of heat output from the working fluid during d→d'→a. Hence or otherwise, write down an expression for the thermal efficiency of this ORC.

[10 marks]

(ii) Briefly explain why it is appropriate to model c→d as an adiabatic process.

[5 marks]

(iii) Write down an expression for the Carnot efficiency corresponding to this cycle.

[5 marks]

7. iPhone 6s has a 12-megapixel camera with a five-element lens for its primary (rear) camera. Below are some of its reported characteristics.

Property	Value
Focal length	4.15 mm
Aperture	*f* /2.2
Sensor array	4032 x 3024 pixel format
	4.92 x 3.69 mm sensor size
	1.22 μm pixel size

(i) This camera is used to first image an object 5.0 m away, and then a board with writings 1.0 m away. Compute the change in image distance that the auto-focus needs to perform in the camera.

[4 marks]

(ii) The writings on the board cover an area of 1.0 m x 0.7 m. Compute whether the sensor array is able to completely capture these writings.

[4 marks]

(iii) Compute the diffraction-limited spot size on the image plane for green light.

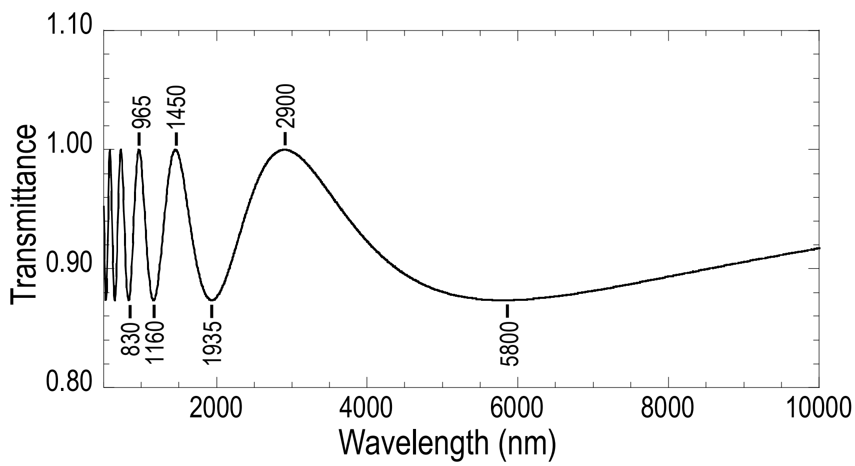
[4 marks]

(iv) Hence or otherwise, estimate the smallest gap between two adjacent ink strokes on the board that can be resolved by the sensor array, assuming the optics are diffraction-limited. Briefly explain your reasoning.

[4 marks]

(v) Briefly explain one aberration that the five-element lens may have been designed to correct. [4 marks]

8. A thin film of silica (SiO_2) with thickness *d* is suspended in air using pillars. Its transmittance spectrum measured at normal incidence as a function of the wavelength from 500 nm to 10,000 nm is shown below, with the first few peaks and valleys marked. Transmittance is the fraction of incident power transmitted through the film. A transmittance of 1 means all the incident power is transmitted. The silica film is completely transparent, and has a constant refractive index *n* of 1.45 over the measured wavelength range.



(i) Briefly explain why transmittance valleys appear in the spectrum. Derive an expression for the wavelength of these valleys λ_{min} using *n*, *d* and any other necessary parameter.

[8 marks]

(ii) Hence or otherwise, evaluate the thickness of the silica film.

[4 marks]

(iii) Briefly explain why perfect transmission is possible at the peaks.

[4 marks]

(iv) Briefly explain the appearance of this transmittance spectrum if the silica film is uniformly thinned down by etching.

[4 marks]

---END OF PAPER ---

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