NATIONAL UNIVERSITY OF SINGAPORE

PC1142 Introduction to Thermodynamics and Optics

(Semester I: AY 2017–18)

Time Allowed: 2 Hours

<u>INSTRUCTIONS TO CANDIDATES</u>

1. Please write your matriculation number in the answer booklet. Do not write your name.
2. This examination paper comprises 9 printed pages with 5 short questions in Part I and 3 long questions in Part II.
3. Answer <u>all</u> questions.
4. This is a <u>closed book</u> examination.
5. You may use electronic calculators.
6. A list of physical constants and formulae is given on pages 2 and 3.

Thermal physics and kinetic theory of gases:

(i) Kinetic mean free path is given by $\ell = \frac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\frac{3k_B T}{m}}$

(b) Average speed: $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$

(c) Most-probable speed: $v_{mp} = \sqrt{\frac{2k_B T}{m}}$

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T$$

(v) One form of adiabat: $p \cdot V^{\gamma} = constant$

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$

(vii) Planck radiation equation:

$$I_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)}$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3} m \cdot K}{T}$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \frac{\Delta T}{L}$

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$

(ii) Carnot heat-engine efficiency $e_C = 1 - \frac{T_C}{T_H}$

(iii) Entropy $dS = \frac{dq_{rev}}{T}$

(iv) Enthalpy $H = E + PV$

(v) Helmholtz free energy $F = E - TS$

(vi) Gibbs free energy $G = H - TS$

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$,

where $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(b) Refracting-surface equation:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \frac{d}{n}$

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}}$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}}$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$

(iv) *f*-number, $f/\# = \frac{f}{D}$

(v) Numerical aperture: $NA = n \cdot \sin\theta$

(vi) Snell's law: $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$

(a) Critical angle: $\sin \theta_c = n_2 / n_1$

(b) Brewster angle: $\tan \theta_p = n_2 / n_1$

(vii) Wave relation: $v = f \cdot \lambda$

(viii) Abbe number: $v = \frac{n_D - 1}{n_F - n_C}$

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta = \frac{1.22\lambda}{a}$

(ii) Slit: first diffraction minimum is at $\sin\theta = \frac{\lambda}{a}$

(iii) *N*-slit intensity pattern: $I = I_o \cdot \frac{\sin^2(N\cdot\phi/2)}{\sin^2(\phi/2)}$, where $\phi = \frac{2\pi}{\lambda} \cdot d \cdot \sin\theta$

(iv) Single-slit diffraction pattern:

$I = I_o \cdot \frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta$

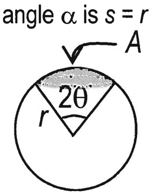
(v) Thin-film interference:

$\delta = 2 \cdot n \cdot d \cdot \cos\beta$, where β is the refracted angle.

General:

(i) Geometry:

The arc length on a circle of radius *r* subtended by angle α is $s = r\,\alpha$



The surface area *A* subtended by polar angle 2θ is:

$A = 2\,\pi r^2 \,(1 - \cos\theta)$

(ii) Logarithms and exponents :

$\log_a (b\, c) = \log_a b + \log_a c$

$\log_a b = \log_d b / \log_d a$

$a^b * a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

(iii) Integrations:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$,

except for $n = -1$, where $\int x^{-1} dx = \ln x + c$

(iv) Taylor expansions:

$\sin\theta = \theta - \frac{1}{6}\theta^3 + \ldots$

$\cos\theta = 1 - \frac{1}{2}\theta^2 + \ldots$

$\tan\theta = \theta + \frac{1}{3}\theta^3 + \ldots$

$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \ldots$

(v) Series sum:

$\sum_{n=0}^{n=m} a_0 r^n = \frac{a_o(1-r^{m+1})}{1-r}$

(vi) Differentiations:

$d(\sin u) = \cos u \, du$

$d(\cos u) = -\sin u \, du$

$d(u^m) = m u^{m-1} \, du$

Universal constants:

Gas constant R = 8.314 J K^{-1} mol^{-1}

Boltzmann constant k_B = 1.381x10^{-23} J K^{-1}

Stefan–Boltzmann constant σ = 5.670x10^{-8} W m^{-2} K^{-4}

Speed of light in vacuum c = 2.998x10^{8} m s^{-1}

Avogadro's number N_A = 6.022x10^{23} mol^{-1}

Permittivity of free space ε_0 = 8.854x10^{-12} Fm^{-1}

Planck constant h = 6.626x10^{-34} J s

Part 1. Answer **ALL** questions. Each question carries 8 marks.

1. The specific heat capacities of dry air at 1000 K are given below.

Property	$\mathrm{kJ\ kg^{-1}\ K^{-1}}$
C_p	1.142
C_v	0.855

(i) Compute the change in specific entropy of dry air when heated at constant volume from 500 K to 1500 K, assuming that its heat capacity over this range is well approximated by the 1000-K value.

(ii) Evaluate the heat capacity ratio γ, and briefly explain why it is different from the "idealized" value of 1.40 for a diatomic gas.

[8 marks]

2. A solid metal sphere with radius r_o is embedded in an infinite medium with thermal conductivity k_m. The metal is heated internally and uniformly to temperature T_o. Assuming that temperature of the medium far away from the sphere is T_m, show that the steady-state heat power loss from this metal sphere is given by $P_c = 4\pi k_m r_o (T_o - T_m)$. Hence or otherwise, write down the expression for the spreading thermal conductance L for this configuration, where $P_c = L(T_o - T_m)$.

[8 marks]

3. Energizer® alkaline batteries produce electrical power from the following reaction.

$$Zn\ (s) + 2\ MnO_2\ (s) + H_2O\ (\ell) \rightarrow ZnO\ (s) + 2\ MnOOH\ (s)$$

The standard enthalpy of formation $\Delta H^o_{f,298}$, Gibbs free energy of formation $\Delta G^o_{f,298}$, and entropy S^o_{298} of these substances at 298 K and 1 bar are given below.

Quantity	Zn (s)	MnO_2 (s)	H_2O (ℓ)	ZnO (s)	MnOOH (s)
$\Delta H^o_{f,298}$ (kJ mol^{-1})	0	–520.0	–285.8	–348.3	n.a.
S^o_{298} (J K^{-1} mol^{-1})	41.6	53.0	69.9	43.6	n.a.
$\Delta G^o_{f,298}$ (kJ mol^{-1})	0	–465.2	–237.2	–318.3	–557.8

Note: n.a. means "not available"

(i) Compute the maximum amount of electrical work that can be obtained per mole of Zn consumed.

(ii) Explain using the Second Law of Thermodynamics why this is the maximum.

[8 marks]

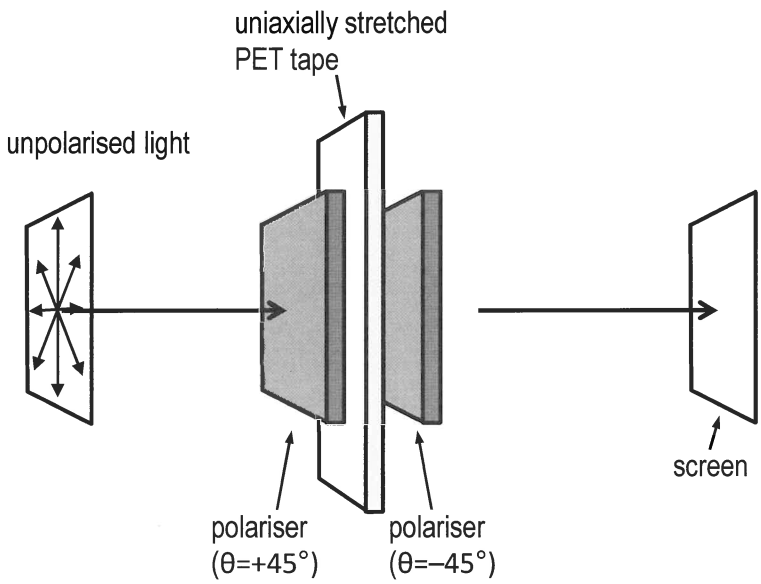
4. An aquarium viewer offered for sale comprises an acrylic plano-convex lens that can be mounted flat onto the acrylic wall of an aquarium tank. The convex side of the lens with radius of curvature R is in contact with air, while the flat side is in contact with the acrylic wall. The refractive indices of water, acrylic and air are n_1, n_2 and n_3, respectively. Assume that the thickness of the acrylic wall is negligible.

(i) Derive an expression in the thin-lens approximation that relates image distance q to object distance p for an object inside the water of the aquarium.

(ii) Hence or otherwise, derive an expression for the angular magnification of this viewer.

[8 marks]

5. A narrow strip of transparent poly(ethylene terephthalate) (PET) tape that is uniaxially stretched along its length is placed between two crossed polarisers that are oriented at +45° and –45° relative to the stretched tape axis, as shown below. The refractive indices of the PET tape are $n_{//}$ = 1.70 and $n_{\perp}$ = 1.50 for visible light, parallel and perpendicular to the stretching direction, respectively. An unpolarised visible light beam is incident normally on the stack.



(i) Briefly explain the image that one may expect to observe on the screen.

(ii) Briefly explain the appearance of this image as the pair of polarisers is rotated in unison about direction of propagation of the light.

[8 marks]

Part 2. Answer **ALL** questions. Each question carries 20 marks.

6. The Atkinson cycle produces better fuel economy than the Otto cycle but with a lower power per unit displacement of the piston. For a hybrid electric vehicle, an electric motor is operated to provide the needed boost when demand for power increases. One variant of an ideal Atkinson cycle consists of:

a*→*b: adiabatic compression of air;

b*→*c: isochoric heating due to fuel combustion;

c*→*d: isobaric heating due to continued fuel combustion;

d*→*e: adiabatic expansion of the hot gas;

e*→*f: isochoric cooling;

f*→*a: isobaric cooling.

(i) Briefly explain the meaning of an adiabatic process in thermodynamics.

[4 marks]

(ii) Sketch this cycle on a pressure–volume (*P*–*V*) state diagram. Label all points ***a*** to ***f*** on this sketch. State the segments that correspond to heat input, power stroke and heat output. [8 marks]

(iii) Write down an expression for the segment ***b*→*c*→*d*** and for the segment ***e*→*f*→*a*** in terms of the relevant temperatures and any other necessary parameters. Hence or otherwise, derive an expression for the thermal efficiency of this cycle in terms of these temperatures. [4 marks]

(iv) Sketch this cycle on a temperature–entropy (*T*–*S*) state diagram. Label all points ***a*** to ***f*** on this sketch. [4 marks]

7. An interstellar asteroid object tentatively named A/2017 U1 was recently discovered with observations made on a Pan-STARRS telescope in Hawaii when that object was 30 million kilometres from Earth. Assuming an albedo of 10%, the object was estimated to have a diameter of 160 m. The telescope has a field-of-view of 3°, and an aperture diameter of 1.8 m. It uses a 1-Gigapixel sensor to image the sky through one of five colour filters. The image sensor consists of an 8 x 8 array of units, each of which further consists of an 8 x 8 array of 512 x 512 pixel cells. The length of each pixel is 10 μm, hence the diameter of the image sensor is 32 cm. The telescope has the Ritchey–Chrétien configuration with hyperbolic primary mirror and hyperbolic secondary mirror to eliminate coma aberrations over the entire field of view.

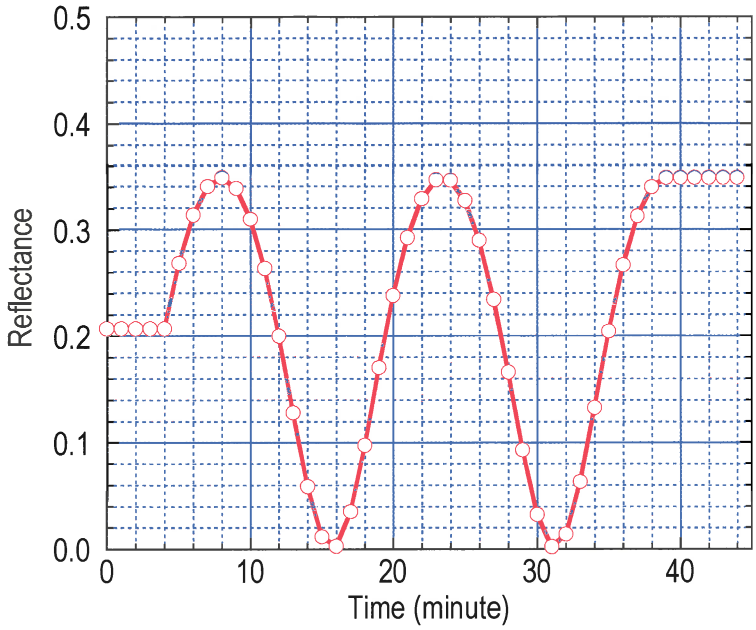
(i) Assuming that the 3° sky view completely fills the image sensor, compute the focal length of this telescope. [4 marks]

(ii) Deduce whether it is possible to evaluate the size of A/2017 U1 directly from the diameter of its image at the diffraction limit using this telescope with red filter (623 nm). [8 marks]

(iii) It has been reported that "the measured point spread function of this telescope is about 2 times larger than the design expectation". Compute the Airy disc diameter and briefly explain the significance of this statement. [4 marks]

(iv) Briefly explain the coma aberration and the benefits of eliminating this in astronomy observations. [4 marks]

8. An overlayer film of silicon nitride (Si_3N_4) on a silicon (Si) substrate was etched (i.e., removed uniformly by forming volatile products) in a plasma chamber operated at low pressure. The Si_3N_4 thickness was monitored by interferometry using 633-nm wavelength at normal incidence to give the reflectance–time plot as shown below. The refractive indices of Si_3N_4 and Si are 2.04 and 3.88, respectively. A reflectance of 1 means all the incident power is reflected. Etching gases were admitted into the chamber at time t = 4.0 min. The Si_3N_4 film was determined to be completely removed at t = 39.0 min.



(i) Briefly explain why the reflectance valleys can have a value close to zero.

[4 marks]

(ii) Derive an equation for the thickness of the Si_3N_4 layer at each reflectance peak.

[8 marks]

(iii) Hence or otherwise, deduce the initial film thickness of the Si_3N_4. [4 marks]

(iv) Sketch and annotate the reflectance–time plot if the etch rate becomes twice as fast. [4 marks]

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