NATIONAL UNIVERSITY OF SINGAPORE

PC1143 INTRODUCTION TO ELECTRICITY & MAGNETISM

(Semester II: AY 2017-18)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name.**
2. This examination paper contains **EIGHT (8)** questions and comprises **SIX (6)** printed pages.
3. Students are required to answer **ALL** questions in Part I and **ALL** questions in Part II.
4. Students should write the answers for each question on a new page.
5. This is a **CLOSED** book examination.
6. The total mark for Part I is 40 and that for Part II is 60.
7. Specific permitted devices: non-programmable calculators.
8. A list of physical constants and mathematical formulae are provided on pages 2 to 3.

Fundamental Physical Constants

Speed of light in vacuum	$c = 2.9979 \times 10^{8}\,\mathrm{m/s}$
Magnitude of charge of electron	$e = 1.6022 \times 10^{-19}\,\mathrm{C}$
Mass of electron	$m_{\mathrm{e}} = 9.1094 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	$m_{\mathrm{p}} = 1.6726 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	$m_{\mathrm{n}} = 1.6749 \times 10^{-27}\,\mathrm{kg}$
Permittivity of free space	$\epsilon_0 = 8.8542 \times 10^{-12}\,\mathrm{C}^2/\left(\mathrm{N}\cdot\mathrm{m}^2\right)$
Coulomb's constant	$k \equiv \dfrac{1}{4\pi\epsilon_0} = 8.9876 \times 10^{9}\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T}\cdot\mathrm{m/A}$
Acceleration due to gravity (standard)	$g = 9.8067\,\mathrm{m/s}^2$

Algebra

$$a^{-x} = \frac{1}{a^x} \qquad a^{x+y} = a^x a^y \qquad a^{x-y} = \frac{a^x}{a^y}$$

If $\log a = x$, then $a = 10^x$. $\quad \log a + \log b = \log(ab) \quad \log a - \log b = \log(a/b) \quad \log(a^n) = n\,\log a$

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$$\text{If } ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Geometry

Circumference of circle of radius r: $C = 2\pi r$

Area of circle of radius r: $A = \pi r^2$

Surface area of sphere of radius r: $A = 4\pi r^2$

Volume of sphere of radius r: $V = 4\pi r^3/3$

Volume of cylinder of radius r and height h: $V = \pi r^2 h$

Volume of cone of radius r and height h: $V = \pi r^2 h/3$

Trigonometry

$$\tan x = \frac{\sin x}{\cos x} \qquad \csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \sin 2x = 2\,\sin x\,\cos x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Calculus

Product rule $$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)\,g(x)\right] = f(x)\,g'(x) + f'(x)\,g(x)$$

Quotient rule $$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$$

Chain rule $$\frac{\mathrm{d}}{\mathrm{d}x}\{f[g(x)]\} = f'\left[g(x)\right]\,g'(x)$$

Fundamental theorem of calculus $$\int_{x=a}^{b} f(x)\,\mathrm{d}x = F(x)\Big|_{x=a}^{b} = F(b) - F(a)\,,\ \frac{\mathrm{d}}{\mathrm{d}x}\left[F(x)\right] = f(x)$$

Integration by parts $$\int_{x=a}^{b} f(x)\,\left[\frac{\mathrm{d}}{\mathrm{d}x}g(x)\right]\,\mathrm{d}x = f(x)\,g(x)\Big|_{x=a}^{b} - \int_{x=a}^{b}\left[\frac{\mathrm{d}}{\mathrm{d}x}f(x)\right]\,g(x)\,\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,x^n = nx^{n-1} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\csc x = -\csc x\,\cot x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\sec x = \sec x\,\tan x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot x = -\csc^2 x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^x = \mathrm{e}^x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}\,, \quad x > 0$$

$$\int x^n\,\mathrm{d}x = \frac{1}{n+1}\,x^{n+1}\,, (n \neq -1) \qquad \int \frac{1}{x}\,\mathrm{d}x = \ln|x| \qquad \int \sin x\,\mathrm{d}x = -\cos x$$

$$\int \cos x\,\mathrm{d}x = \sin x \qquad \int \tan x\,\mathrm{d}x = \ln|\sec x| \qquad \int \csc x\,\mathrm{d}x = \ln|\csc x - \cot x|$$

$$\int \sec x\,\mathrm{d}x = \ln|\sec x + \tan x| \qquad \int \cot x\,\mathrm{d}x = \ln|\sin x \qquad \int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x$$

$$\int \ln x\,\mathrm{d}x = x\,\ln x - x$$

Power Series

Binomial expansion $$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\,x^2 + \frac{n(n-1)(n-2)}{3!}\,x^3 + \cdots \quad (|x| < 1)$$

Taylor expansion $$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}\,(x-a)^2 + \frac{f^{(3)}(a)}{3!}\,(x-a)^3 + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \quad (\text{all } x) \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \quad (\text{all } x)$$

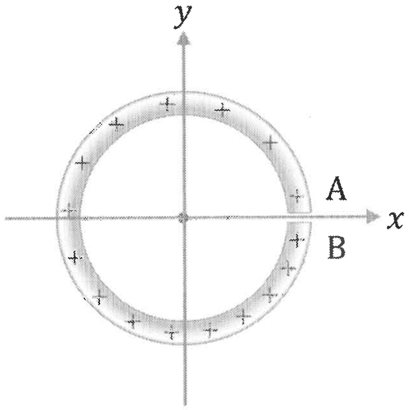
$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \quad (|x| < \pi/2) \qquad \mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad (|x| < 1)$$

PART I

Question 1 **[8=2+6]**

A non-conducting ring of radius R, centered at the origin, lies on the xy plane. The total charge of the ring is Q. The charge on the ring increases linearly from end A to end B at which both ends almost meet very near the right of the ring.



(a) Find the electric potential at the origin by taking the zero of the electric potential at infinity.

(b) Hence, or otherwise, find the electric field at the origin.

Question 2 **[8=5+3]**

The region between concentric spheres is filled with a material with conductivity σ and dielectric constant κ. The inner sphere (radius a) is connected to the positive terminal of a battery with emf V_0 and the outer sphere (radius b) is connected to the negative terminal. At $t = 0$, the battery is disconnected.

(a) Calculate the charge on the inner conductor as a function of time.

(b) Calculate the current flowing between the spheres as a function of time.

Question 3 **[8=4+4]**

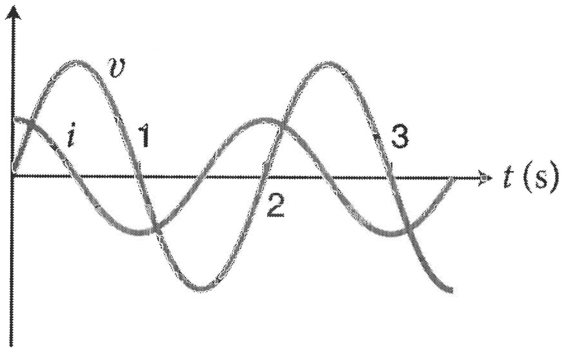
An electron travels to the right at $2.5 \times 10^6\ \mathrm{m/s}$ between two large, flat sheets that are parallel to each other and to the electron's line of motion. A current per unit width of $8.0\ \mathrm{A/m}$ flows to the right through the top sheet.

(a) What current should flow in the second sheet so that the electron does not deviate from its path?

(b) If one generates a current of $10\ \mathrm{A/m}$ to the right on the second sheet, what is the magnitude and direction of the magnetic force on the electron?

Question 4 **[8=1+7]**

You are given an AC circuit to analyze by your laboratory instructor. You are told that the circuit contains a resistor and another unknown circuit element in series with the resistor. Additionally, you know that the circuit element must be one of the following: a 0.32-F capacitor, a 0.32-mF capacitor, a 6.28-mH inductor or a 62.8-mH inductor. The curves for the current through the unknown element and the voltage across it are given below.



The maximum current is $1.00\,\mathrm{mA}$ and the maximum voltage is $1.00\,\mathrm{V}$. Identify the unknown element. Explain clearly your reasoning and substantiate your answer with calculations.

Question 5 **[8=5+3]**

A short wire lies on the x axis extending from $x = -a$ to $x = +a$. At $t = 0$, there is a small sphere carrying charge $-q_0$ at $x = -a$ and another sphere carrying charge $+q_0$ at $x = +a$. As the spheres discharge, a current is established in the wire.

(a) Find the total displacement current through a flat circular surface of radius y_0 in the yz plane centered at the origin.

(b) Hence, find the magnetic field at the point $P(0, y_0, 0)$ where $y_0 > 0$. Indicate clearly the direction of the magnetic field.

END OF PART I

PART II

Question 6 **[20=5+12+3]**

A capacitor consists of three coaxial cylindrical shells with radius R, $2R$ and $3R$ respectively. The inner and outer shells are connected by a wire (passing through a hole in the middle shell *without* touching it). The shells start neutral and then a battery transfers charges from the middle shell to the inner/outer shells. The final charge per unit length on the middle shell is $-\lambda$.

(a) Find the capacitance per unit length of the system by treating it as a simple combination of cylindrical capacitors.

(b) Find the charges per unit length on the inner and outer shells in terms of λ.

(c) Now, the battery is disconnected and additional charge per unit length λ_{extra} is added to the outer shell. What would happen to the three charges per unit length of the shells? Explain your answer.

Question 7 **[20=6+10+4]**

A square wire loop of size $2a \times 2a$ lies in the xy plane with its center at the origin and sides parallel to the x and y axes. A current I flows counter-clockwise around the loop as viewed from the $+z$-axis.

(a) Find the magnetic field at an arbitrary point $P(x_0, y_0, z_0)$ due to the segment of the wire loop extending from $(a, -a, 0)$ to $(a, a, 0)$. Leave your answer in terms of a definite integral.

(b) Hence, or otherwise, find the magnetic field at a distance z_0 above the center of the square loop.

(c) Show that for $z_0 \gg a$ the magnetic field of this square loop becomes that of a magnetic dipole and find the magnetic dipole moment.

Question 8 **[20=10+4+6]**

A long cylindrical solenoid of radius R is centered on the z axis extending from $z = -L/2$ to $z = +L/2$. It has N turns and there is a current $I(t) = \alpha t$ through the turns where α is a positive constant. The current flows counter-clockwise around the solenoid as viewed from the $+z$-axis.

(a) Calculate the Poynting vector at $r = R$ from the center of the cylindrical axis of the solenoid. Indicate clearly the direction of the Poynting vector.

(b) Using your expression for the Poynting vector, derive an expression for the power input to the solenoid as a function of time.

(c) Calculate the power from the stored magnetic energy. Does this answer agree with your answer above? Explain.

END OF PART II

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END OF PAPER