

1. (a) Obviously, the potential outside a sphere is $V = \frac{Q}{4\pi\epsilon_0 r}$.

$$Q = \frac{3Q^2 r_1^4}{4\pi\epsilon_0 r^6} dr_1$$

(b)

- (c) Integrating the expression above and setting $r = R, r_1 = r$,

$$W = \frac{4\rho^2 \pi R^5}{15\epsilon_0}$$

2. (a) A steady voltage is built up when the Lorentz force and electric force acting on the charge carriers passing through the conductor. Hence,

$$\Delta V_H = \frac{IBd}{nqA}$$

(b) The Hall Effect is helpful in determining the sign of majority charge carriers in an unknown material. By keeping I and B in the expression above constant, the sign of the Hall voltage changes if the sign of q changes. Hence, we could know the sign of the majority charge carriers in the material by measuring the Hall voltage.

3. By inspecting the circuit, we could know that those resistors at upper left and upper right corners are in parallel. Assume the current flowing in the battery X.

By Kirchoff's law, using the left part of the circuit and going anticlockwise,

$$X = 186.0V$$

- i. The emf of X is 186.00V
 ii. The current I through 200.0V battery is 3.00A flowing upwards.
 iii. R=20.0 ohm.
4. (a) For electric field, $\oint_s \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$
 For magnetic field, $\oint_s \vec{B} \cdot d\vec{A} = 0$

(b) The net magnetic flux passing through a closed surface is zero, which means that there is no magnetic monopole.

(c) The electric field due to a spherically symmetrical charge distribution must be radial; if it had tangential component, we could tell the difference of its electric field as the charge distribution was rotated, destroying its symmetry. Hence, for a spherically symmetrically charge distribution, the electric field must be radial.

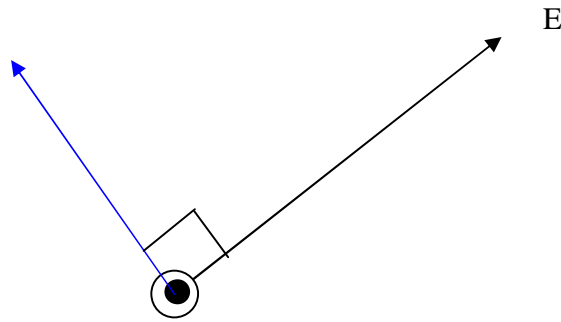
(d) Inside the shell, the enclosed charge $Q=0$, hence

$$E = 0$$

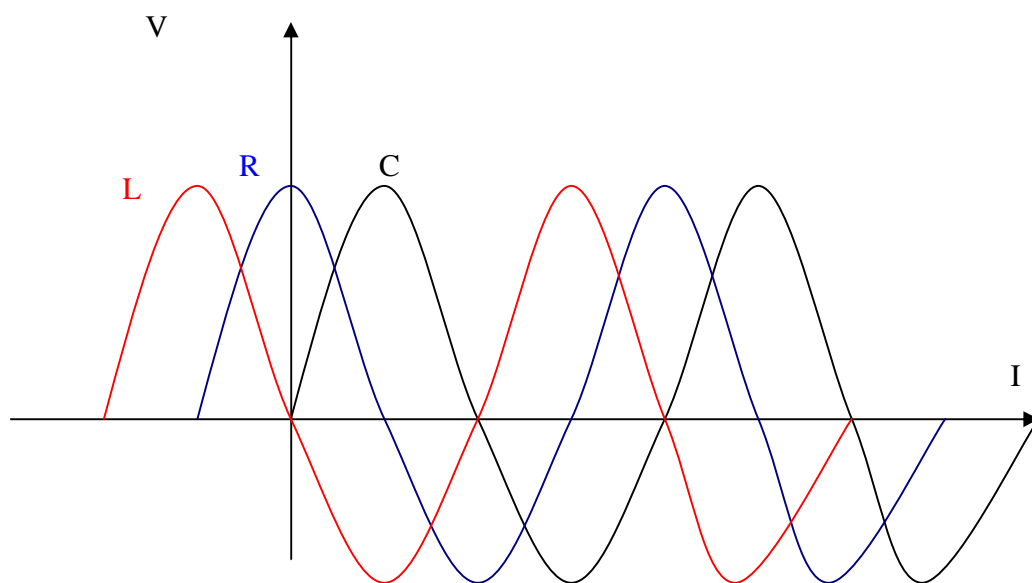
Outside the shell,

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

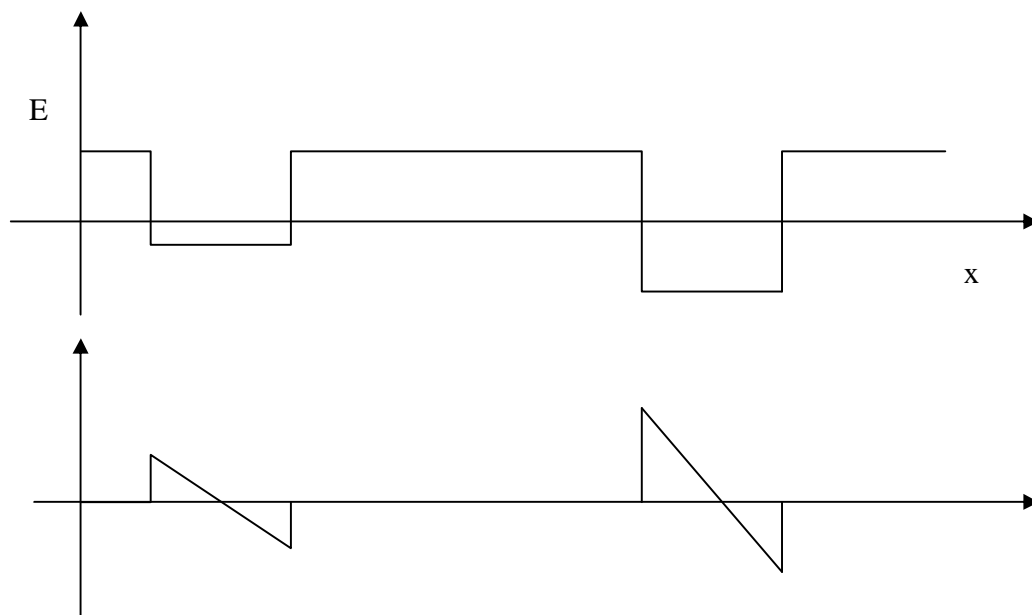
5. (a)



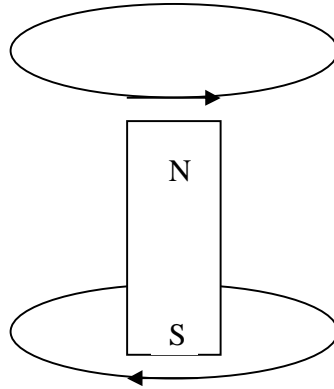
(b)



(c)



(d)



6. (a) The vertical components of the electric field due to the ring cancel out each other, hence the vertical component is zero.

$$E_x = \frac{Qx}{4\pi\epsilon_0\sqrt{(x^2 + a^2)^3}}$$

- (b) Assumptions: The ring has negligible thickness, and is uniformly charged.

$$V = \frac{Q}{4\pi\epsilon_0\sqrt{(x^2 + a^2)}}$$

- (c) The work done in moving a test charge against the electric field will add in its potential energy. Thus, the electric force is doing *negative* work.

$$V = -\int E dx$$

- (d) If the potential is zero, that means the work done in moving a test charge from infinity to that point is zero. However that doesn't indicate the electric field at that particular point is zero as well. For example, the potential at the midpoint between a electric dipole is zero, but there is electric field pointing from positive charge to negative charge.

(e)

$$E = \frac{Qx}{4\pi\epsilon_0\sqrt{(x^2 + a^2)^3}}$$

(f) As the electric field is radial outwards, a positive charge is unstable if placed at the centre of the ring.

(g) Maximum speed is achieved when all its potential energy is converted into its kinetic energy.

$$v = \sqrt{\frac{2Qq}{4\pi\epsilon_0 ma}}$$

(h) An accelerating charge will emit electromagnetic wave, which carries away energy. Hence, the actual speed of the charge is smaller than that calculated in (g).

7. (a) $\langle u \rangle$ is determined as shown.

$$\begin{aligned} &= \frac{\epsilon_0 E_{\max}^2}{T} \left[\frac{\sin 2(kx - \omega t)}{-2\omega} + t \right]_0^T \\ &= \frac{1}{2} \epsilon_0 E_{\max}^2 \end{aligned}$$

$$\text{Similarly, } \langle u \rangle = \frac{1}{2} \frac{B_{\max}^2}{\mu_0}$$

(b) $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ It represents the energy per unit time, per unit area, transported by the fields. Its direction is the direction of the electromagnetic wave.

(c) The average power per unit area transported by an electromagnetic wave is called the intensity. $I = \langle S \rangle$

(d) If a point has constant power, as we move away from it, the power per unit area is decreased because the power has to be divided equally in bigger surface area.

(e)

$$\begin{aligned}P &= IA \\ &= I(4\pi r^2) \\ I &= \frac{P}{4\pi r^2}\end{aligned}$$

(f)

$$p = \frac{2I}{c}$$

(g)

$$F = \frac{AP}{2\pi r^2 c}$$

(h)

$$W = \frac{AP}{2\pi c} \left(\frac{1}{R_E} - \frac{1}{r} \right)$$

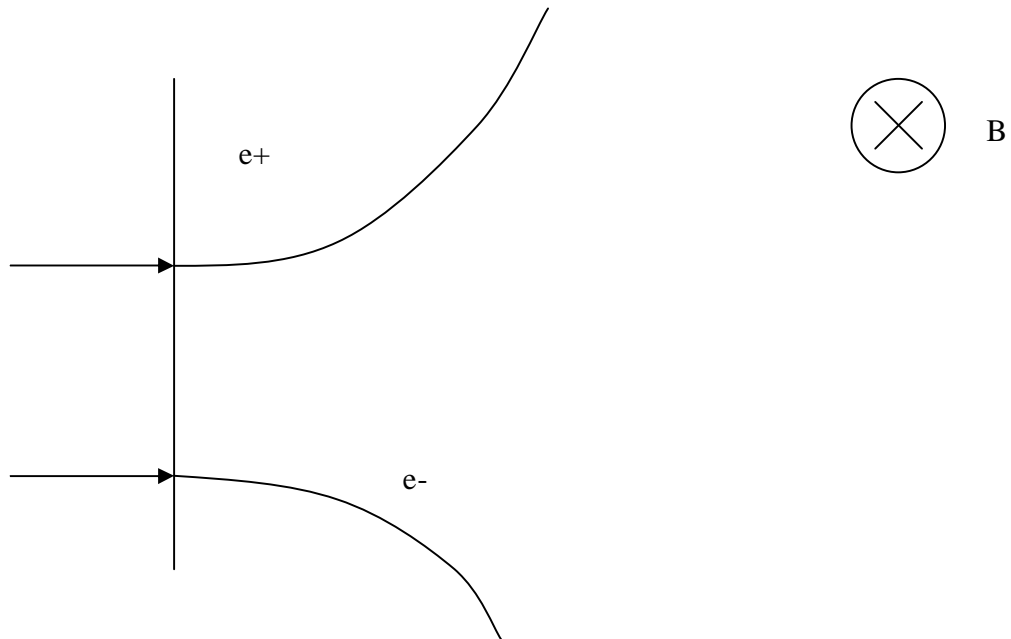
By conservation of energy,

$$t = \int_{R_E}^{R_S} \frac{dr}{\sqrt{\frac{AP}{M\pi c} \left(\frac{1}{R_E} - \frac{1}{r} \right)}}$$

8. (a) $\vec{F} = q(\vec{v} \times \vec{B})$

(b) From the expression above, we could see that all the three vectors F , v and B are orthogonal to each other, hence the force acts perpendicularly to the velocity of the charged particle, and hence no work is done by the magnetic force.

(c)



(d) The particles lose energy because they attract each other, causing the trajectory to change.

(e) We know the force acting on a charge is given by $\vec{F} = q(\vec{v} \times \vec{B})$. Hence, the force acting on a segment of a wire which consists of n number of charge carriers is

$$\begin{aligned}\vec{F} &= Nq(\vec{v} \times \vec{B}) \\ &= I(\vec{L} \times \vec{B})\end{aligned}$$

where $I = nAqv$

$$\begin{aligned}&= \frac{N}{AL} Aqv \\ &= \frac{N}{L} qv\end{aligned}$$

F is the force, I is the current flowing, L is the length of the segment of the wire and B is the magnetic field strength.

(f) The magnetic force doesn't do work; it only deflects the charge only. In this case, the electrons in the wire are deflected downwards, building up a voltage across the wire. It is the electrostatic force that is doing work, pulling the wire upwards against gravity.

(g) Once the whole circuit has entered the magnetic field, the system will not move upwards because as the upper and lower part of wire enter the magnetic field, the Lorentz force acting on the circuit will cancel out each other, so the whole circuit will fall.