NATIONAL UNIVERSITY OF SINGAPORE

PC1143 PHYSICS III

(Semester II: AY 2007-08)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **5 short** questions in Part I and **3 long** questions in Part II. It comprises **9** printed pages.
2. Answer **ALL** the questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The total marks for Part I is 40 and that for Part II is 60.
6. A table of constants and mathematical formulae is attached.

PART I

This part of the examination paper contains **five** short-answer questions on pages 2 to 3. Answer **all** questions.

Question 1

A rod of length L lies perpendicular to an infinitely long, uniform line charge of charge density λ C/m (Figure 1).

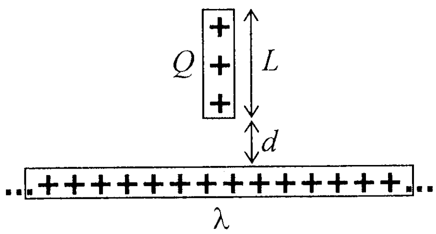

Figure 1

The near end of the rod is a distance d above the line charge. The rod carries a total charge Q uniformly distributed along its length. Find the magnitude of the force that the infinitely long line charge exerts on the rod. [8]

Question 2

Consider the circuit in Figure 2,

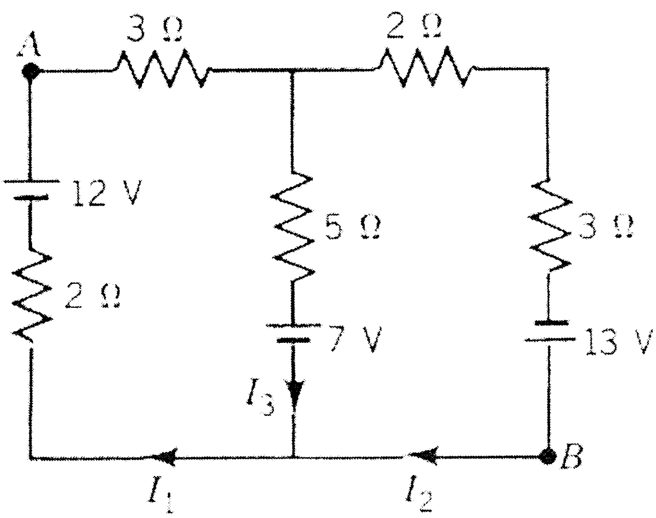

Figure 2

Find

(a) the currents I_1, I_2, and I_3. [6]

(b) the potential difference $V_A - V_B$. [2]

Question 3

A solenoid consists of a wire of radius a wrapped in a single layer on a paper cylinder of radius r. Show that the time constant is

$$\tau \equiv \frac{L}{R} = \frac{\mu_0 \pi a r}{4\rho}$$

where ρ is the resistivity. [8]

Question 4

In the R-L-C circuit of Figure 3, take $R = 8.00\ \Omega$, $L = 40.0\ \text{mH}$, $C = 20.0\ \mu\text{F}$.

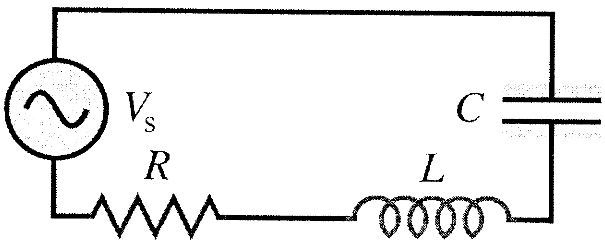

Figure 3

The ac source has voltage amplitude $V = 100\ \text{V}$, and frequency $f = 200/\pi\ \text{Hz}$. Find

(a) the impedance of the circuit. [2]

(b) the peak current in the circuit. [1]

(c) the peak potential difference across R, L, and C. [3]

(d) the peak potential difference across R and L combined. [2]

Question 5

The electric field of a plane travelling electromagnetic wave is given by

$$E_x = E_{\max}\cos(kz + \omega t),\ E_y = E_z = 0.$$

(a) State the direction of propagation. [1]

(b) Determine the magnitude and direction of $\vec{B}$. Express the maximum value of $\vec{B}$ in terms of $E_{\max}$. [3]

(c) Show that the average intensity – that is, the average rate of energy transport per unit area – of the above electromagnetic wave is given by

$$S_{\text{av}} = \frac{cB_{\max}^2}{2\mu_0}.\ [4]$$

END OF PART I

PART II

This part of the examination paper contains **three** long questions on pages 4 to 9. Answer **all** questions.

Question 6(a)

A very long cylinder of radius R_1 and uniform charge density ρ has a cylindrical hole cut along its entire length so that the axes of the cylinder and the hole are parallel and separated by a distance d (Figure 4).

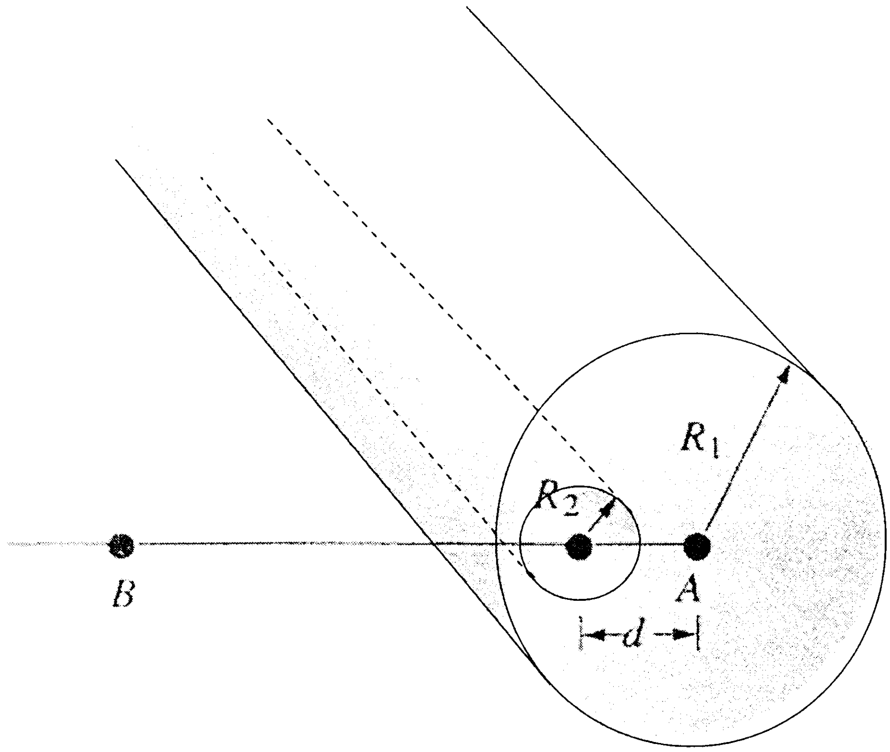

Figure 4

The radius of the hole is $R_2 < R_1$. Compute the electric field strength along the line AB in the following regions:

(i) $r \le d - R_2$.

(ii) $d - R_2 \le r \le d + R_2$.

(iii) $d + R_2 \le r \le R_1$. [8]

Question 6(b)

A spherical capacitor consists of two concentric, metallic spheres, as shown in Figure 5

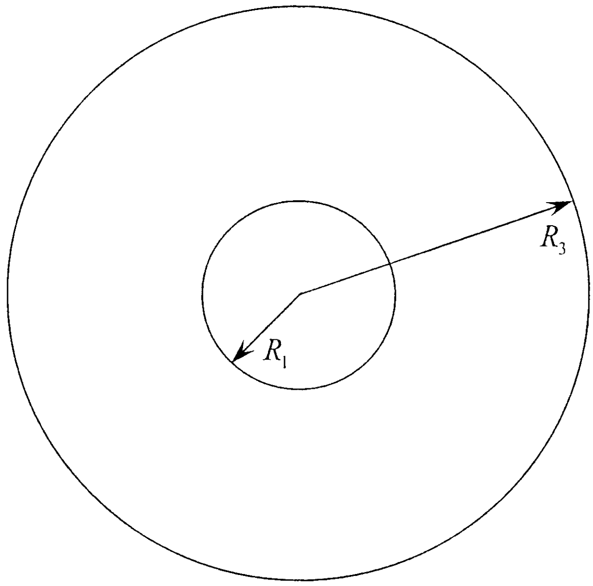

Figure 5

The inner sphere, of radius R_1, has charge $+Q$. The charge on the outer shell of radius R_3 $(R_1 < R_3)$ is $-Q$.

(i) Suppose the spheres are separated by a vacuum, find the capacitance. [6]

(ii) Now, suppose between the spheres are two concentric, spherical dielectrics of constants K_1 and K_2. As shown in Figure 6, the radius of the boundary between the two dielectrics is R_2 $(R_1 < R_2 < R_3)$.

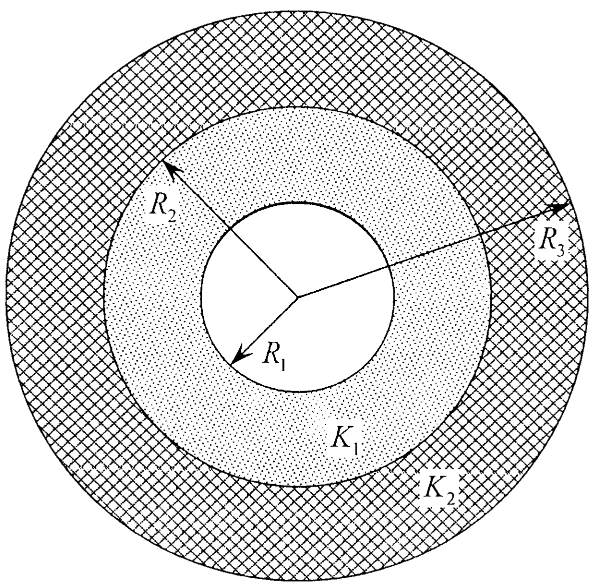

Figure 6

Compute the capacitance of the arrangement. [6]

Question 7(a)

Consider a thin, straight wire of length $2a$, carrying a constant current I, and placed along the x axis as shown in Figure 7.

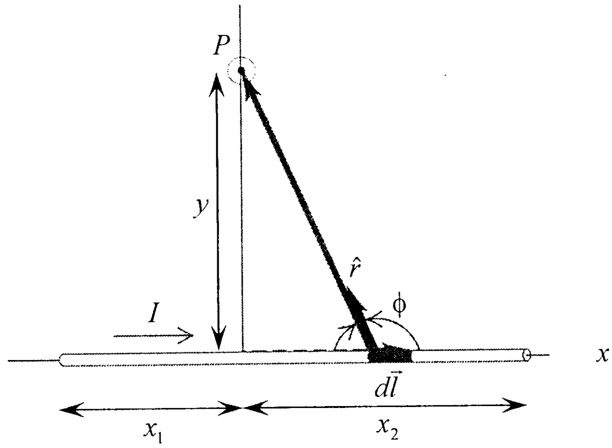

Figure 7

Show that the magnitude of the magnetic field at point P due to this current is given by

$$B = \frac{\mu_0 I}{4\pi y}\left(\frac{x_1}{\sqrt{x_1^2 + y^2}} + \frac{x_2}{\sqrt{x_2^2 + y^2}}\right). \quad [7]$$

Question 7(b)

Compute the magnetic field strength at point P shown in Figure 8 in terms of the radius a, the length b, and the current I. [7]

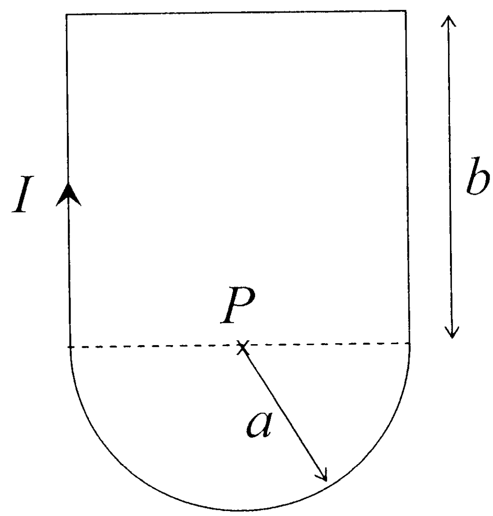

Figure 8

Question 7(c)

A long, cylindrical conductor of radius b has a cylindrical cavity of radius a running parallel to its axis. The axis of the cavity is located a distance d from the axis of the cylinder as shown in Figure 9.

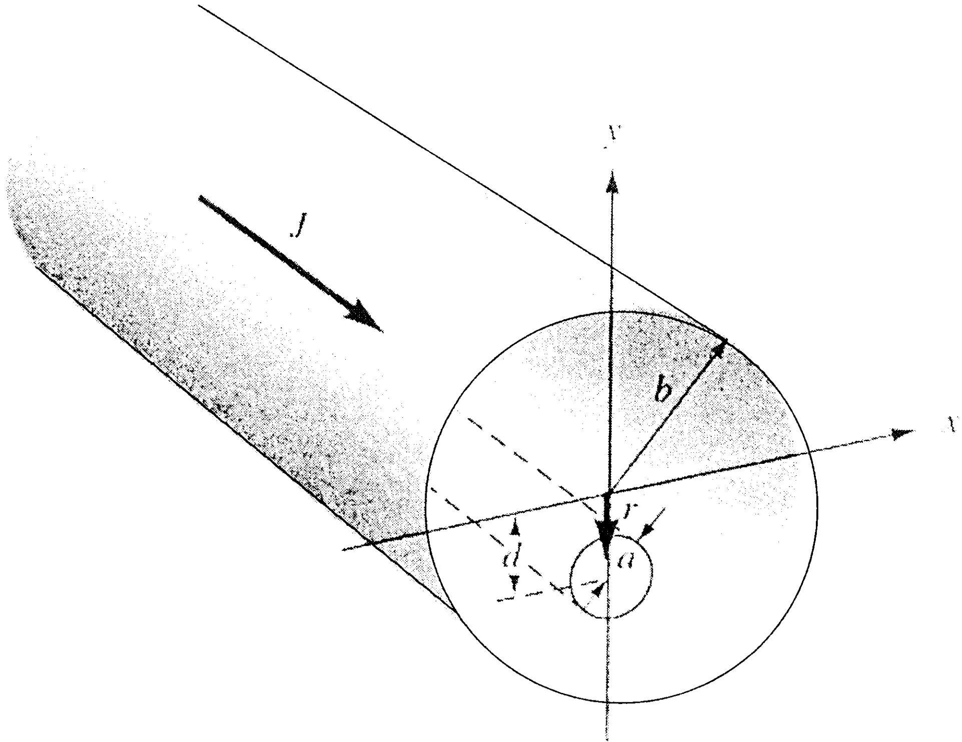

Figure 9

If the conductor carries a uniform current density J, compute the magnitude of the magnetic field strength within the cavity. [6]

Question 8(a)

A rectangular loop of wire with dimensions l and w is dropped at $t = 0$ from rest just above a region in which the magnetic field is B_0 as shown in Figure 10.

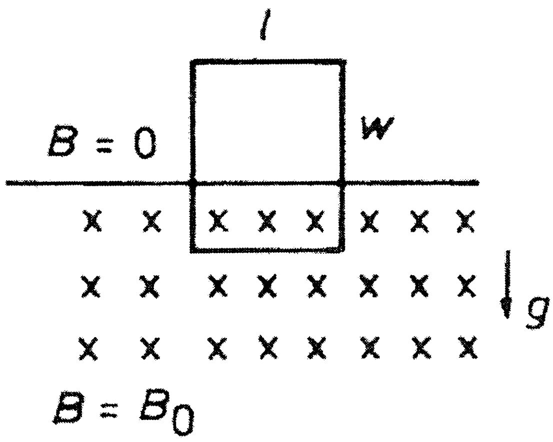

Figure 10

The loop has resistance R, self-inductance L, and mass m. Consider the loop during the time that it has its upper edge in the region of zero magnetic field.

(i) Show that

$$mgv = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + i^2R,$$

where v is the instantaneous speed of the wire loop and i is the instantaneous current in it. [2]

(ii) Show that v satisfies

$$m\frac{dv}{dt} = mg - B_0 il. \quad [2]$$

(iii) Assume that the self-inductance can be ignored but not the resistance, find the current i and speed v as functions of time t. [8]

Question 8(b)

Figure 11 shows a circular parallel-plate capacitor being charged.

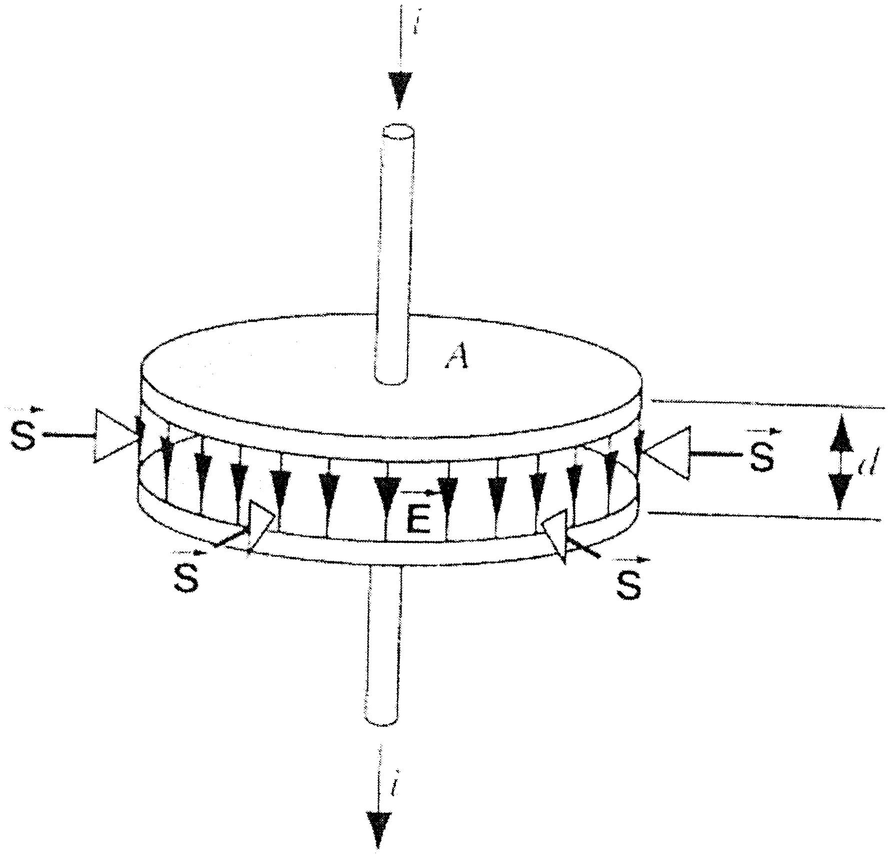

Figure 11

(i) Show that the Poynting vector $\vec{S}$ points everywhere radially into the cylindrical volume. [2]

(ii) Show that the rate at which energy flows into this volume, calculated by integrating the Poynting vector over the cylindrical boundary of this volume, is equal to the rate at which the stored electrostatic energy increases; that is

$$\int \vec{S} \cdot d\vec{A} = Ad\frac{d}{dt}\left(\frac{1}{2}\varepsilon_0 E^2\right). \quad [6]$$

Ignore fringing of $\vec{E}$.

(Y.Y)

END OF PART II

END OF PAPER

A. Fundamental Physical Constants

Speed of light, $c \approx 2.998 \times 10^8$ m/s

Magnitude of charge of electron, $e \approx 1.602 \times 10^{-19}$ C

Mass of electron, $m_e \approx 9.109 \times 10^{-31}$ kg

Mass of proton, $m_p \approx 1.673 \times 10^{-27}$ kg

Permitivity of free space, $\epsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{C^2 \cdot N^{-1} \cdot m^{-2}}$

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb \cdot A^{-1} \cdot m^{-1}}$

Acceleration due to gravity, $g \approx 9.807\ \mathrm{m/s^{-2}}$

B. Derivatives

$$\begin{aligned}
\frac{d}{dx} x^n &= n x^{n-1} \\
\frac{d}{dx} \sin ax &= a \cos ax \\
\frac{d}{dx} \cos ax &= -a \sin ax \\
\frac{d}{dx} e^{ax} &= a e^{ax} \\
\frac{d}{dx} \ln ax &= \frac{a}{x}
\end{aligned}$$

C. Power series

$$\begin{aligned}
(1+x)^n &= 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \cdots \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots
\end{aligned}$$

$$
\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots
\end{aligned}
$$

D. Integrals

$$
\begin{aligned}
\int x^n dx &= \frac{x^{n+1}}{n+1} \ (n \neq -1) \\
\int \frac{dx}{x} &= \ln x \\
\int \sin ax dx &= -\frac{1}{a} \cos ax \\
\int \cos ax dx &= \frac{1}{a} \sin ax \\
\int e^{ax} dx &= \frac{1}{a} e^{ax} \\
\int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin \frac{x}{a} \\
\int \frac{dx}{\sqrt{a^2 + x^2}} &= \ln(x + \sqrt{a^2 + x^2}) \\
\int \frac{dx}{a^2 + x^2} &= \frac{1}{a} \arctan \frac{x}{a} \\
\int \frac{dx}{(a^2 + x^2)^{3/2}} &= \frac{1}{a^2} \frac{x}{\sqrt{a^2 + x^2}} \\
\int \frac{x dx}{(a^2 + x^2)^{3/2}} &= -\frac{1}{\sqrt{a^2 + x^2}}
\end{aligned}
$$