NATIONAL UNIVERSITY OF SINGAPORE

PC1143 PHYSICS III

(Semester II: AY 2011-12)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **5 short** questions in Part I and **3 long** questions in Part II. It comprises **7** printed pages.
2. Answer **ALL** the questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The total marks for Part I is 40 and that for Part II is 60.
6. A table of constants and mathematical formulae is attached.

PART I

This part of the examination paper contains **five** short-answer questions on pages 2 to 3. Answer **ALL** questions.

Question 1

Concentric spherical shells of radius a and b, with $a < b$, carry charge $+Q$ and $-Q$ respectively, each charge uniformly distributed.

(a) Find the energy U_0 stored in the electric field of this system if the shells are separated by vacuum. [6]

(b) Hence, write down the energy U stored in the electric field if the shells are instead being separated by a dielectric with constant $\kappa > 1$. [1]

(c) Is U larger or smaller that U_0? Explain. [1]

Question 2

(a) An electron is moving at a speed $0.01c$ on a circular orbit of radius $10^{-8}\,\text{cm}$. What is the strength of the resulting magnetic field at the centre of the orbit? [4]

(b) Three long straight parallel wires are located as shown in Figure 1.

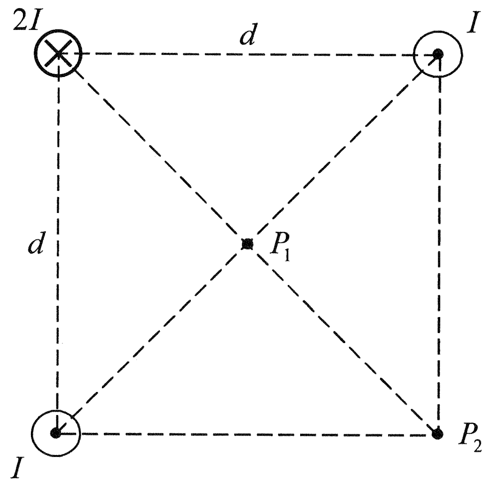

Figure 1

One wire carries current $2I$ into the paper; each of the others carries current I in the opposite direction. Calculate the strength of the magnetic field

(i) at the point P_1? [2]

(ii) at the point P_2? [2]

Question 3
Consider the circuit shown in Figure 2. Assume the switch has been in the position shown long enough that the *steady state* has been reached.

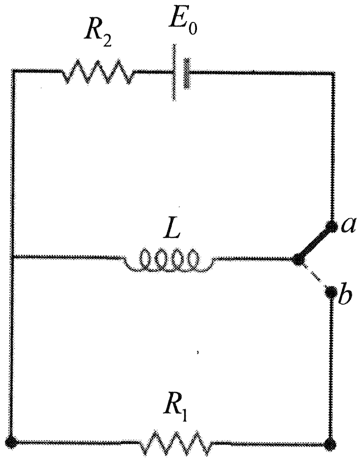

Figure 2

Now at time $t = 0$, flip the switch from a to b.

(a) Determine the current $i(t)$ for time $t > 0$. [4]

(b) Show that the total energy dissipated in the resistor after time $t = 0$ is equal to the energy stored in the inductor at time $t = 0$. [4]

Question 4
A resistor R, inductor L, and capacitor C are connected in parallel to an ac source with voltage amplitude V and angular frequency ω. Suppose the source voltage is given by $v = V \sin \omega t$.

(a) Determine i_R, i_L, and i_C, the currents through the resistor, the inductor, and the capacitor, respectively. [5]

(b) Find the rate at which energy is dissipated in the circuit. [1]

(c) Find the energy is stored in the circuit, i.e., the energy in the capacitor plus the energy in the inductor at any time t. [2]

Question 5
A *plane electromagnetic sinusoidal wave* has the following characteristics. It is travelling along the x-axis in the negative x-direction; its frequency is 100 MHz; the electric field **E** is perpendicular to the z-direction.

(a) Write out formulas for the electric field **E** and magnetic field **B** that specify this wave. [4]

(b) Calculate the *Poynting vector* for this electromagnetic wave. [2]

(c) If the *intensity* of the electromagnetic wave is $1000\,\text{W/m}^2$, calculate the electric and magnetic field amplitudes. [2]

END OF PART I

PART II

This part of the examination paper contains **three** long questions on pages 4 to 7. Answer **ALL** questions.

Question 6(a)

Positive charge Q is *uniformly distributed* along the z-axis between $z=-l$ and $z=+l$, as shown in Figure 3.

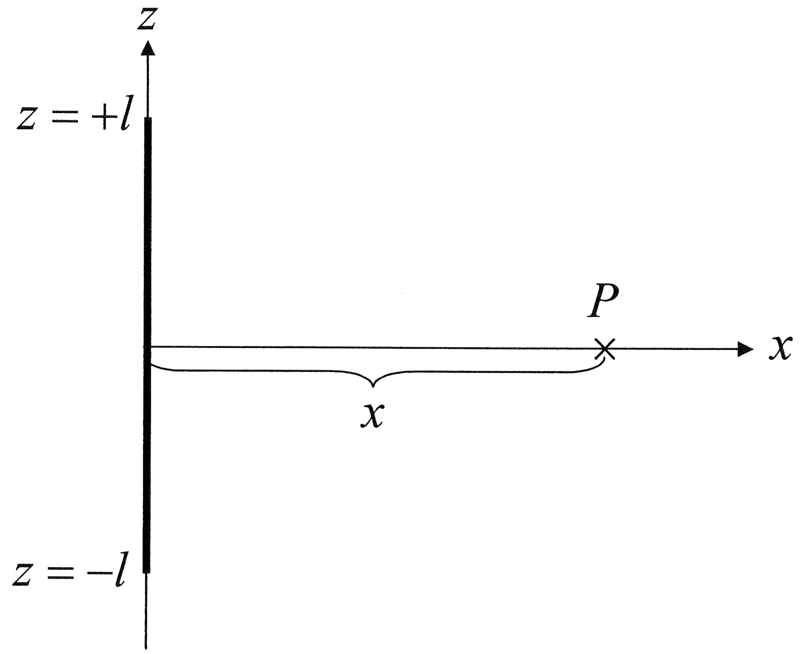

Figure 3

(i) Find the electric field at point P on the x-axis at a distance x from the origin. [6]

(ii) If the *line charge* is infinitely long, find the electric field at P. Assume the charge per unit length remains the same. [2]

Question 6(b)

(i) Find the capacitance of a capacitor that consists of two *coaxial* cylinders, of radii a and b, and length L. Assume $L >> b-a$ so that *end corrections* may be neglected, i.e., you may assume the cylinders to be infinitely long. [8]

(ii) If the gap between the cylinders, $b-a$, is very small compared with the radius a, find the capacitance. Comment on your answer. [4]

Question 7(a)
Consider a circular current loop in the xy-plane, with radius a, centred at the origin O, and carrying current I.

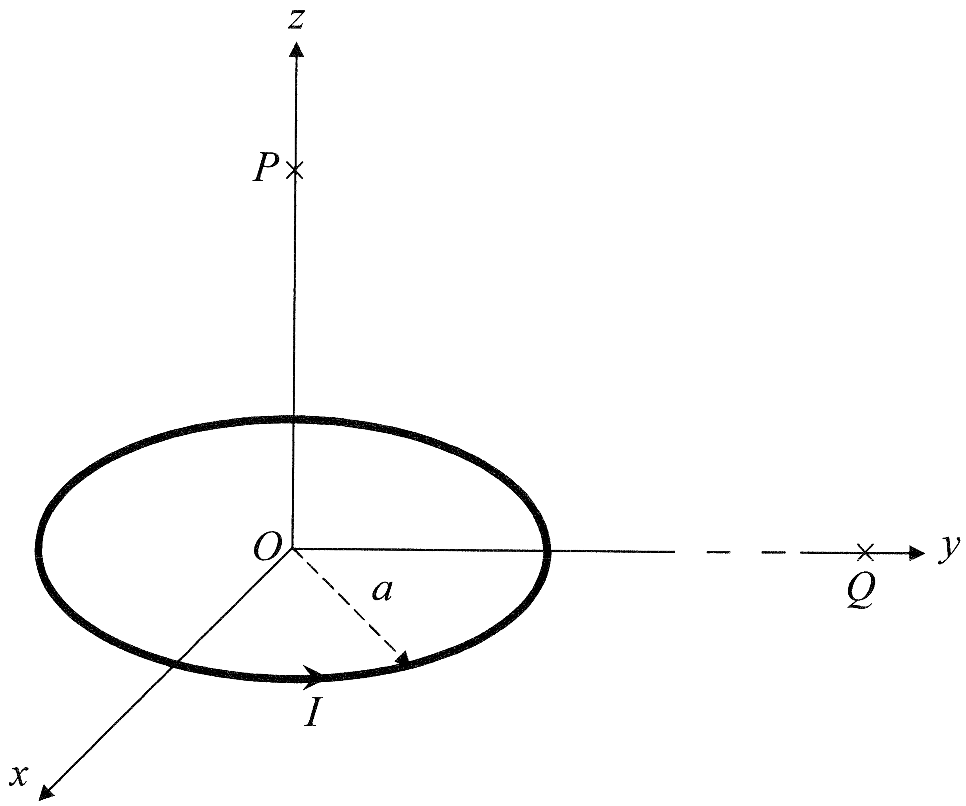

Figure 4

(i) Show that the magnetic field due to this loop at an arbitrary point P on the z-axis – the axis of symmetry – as shown in Figure 4, is given by $\mathbf{B} = B_z\hat{\mathbf{k}}$ where

$$B_z = \frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}.$$

[8]

(ii) Evaluate the integral

$$\lim_{L\to\infty}\int_{-L}^{L} B_z dz\,.$$

[2]

Question 7(b)
Find the magnetic field on the axis of a finite solenoid of radius a centred on the z-axis, extending from $z=-l/2$ to $z=+l/2$, and with n turns per unit length of wire carrying current I. [4]

Question 7(c)
(i) Determine the magnetic field due to the circular loop in Figure 4, at a point Q on the y-axis, far away. [4]
(ii) Hence, explain how *Ampere's law* gives the same answer to the integral in 7(a) (ii). [2]

Question 8(a)

Figure 5 shows a circular wire loop of wire with radius a lying in the xy-plane. The left half has electrical resistance R_1, while the right half R_2.

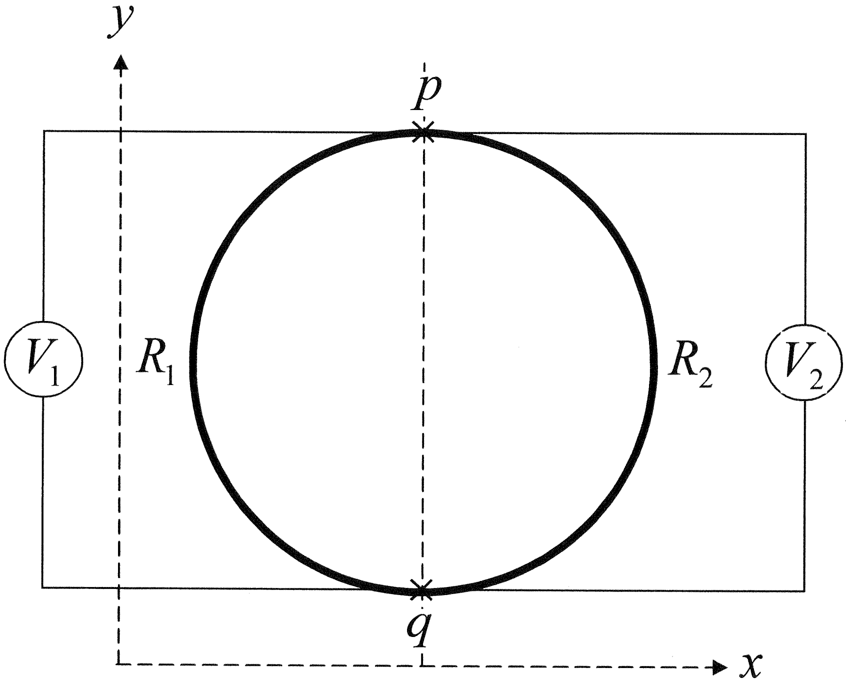

Figure 5

Over the area within the circular wire loop, a uniform magnetic field is turned on at time $t = 0$; for $t > 0$ the field is

$$\mathbf{B}(t) = \frac{B_0}{\sqrt{2}}[1 - \exp(-\lambda t)](\hat{\mathbf{j}} + \hat{\mathbf{k}}),$$

where B_0 and $\lambda > 0$ are real parameters.

(i) Determine the current $I(t)$ induced in the loop for $t > 0$. [5]

(ii) Sketch a graph of the magnitude of $I(t)$ versus t. [3]

(iii)Determine the readings on the two voltmeters V_1 and V_2. [2]

(iv)How is it possible that the readings are different? The readings are different even if $R_1 = R_2$. Should there not be a unique voltage drop between points p and q? Explain. [4]

Question 8(b)
Figure 6 shows a single *equilateral* triangular loop whose *height* is a, and a long straight wire in the same plane.

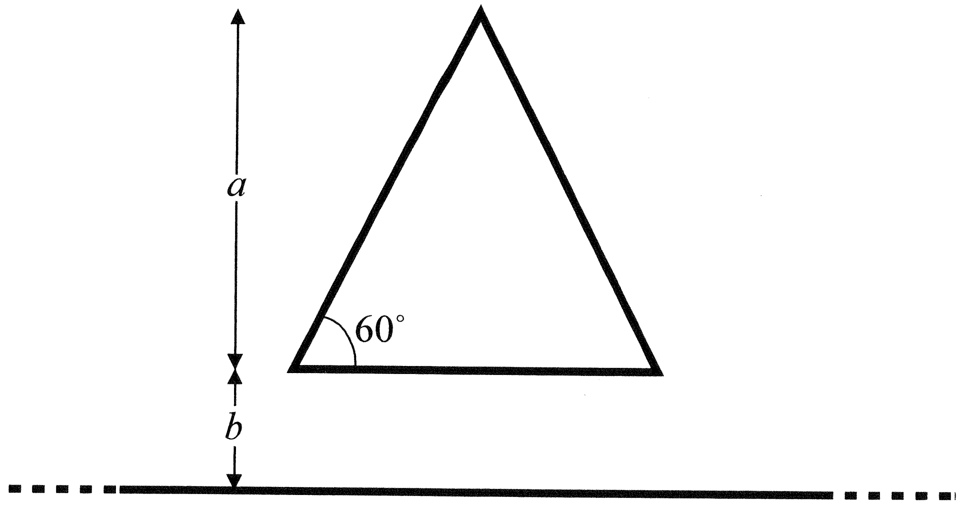

Figure 6

The wire is parallel to and at distance b from the base of the triangle. What is the *mutual inductance*? [6]

Hint: You may find the following result useful.

$$\tan 60^\circ = \sqrt{3}$$

YY

END OF PART II
END OF PAPER

A. Fundamental Physical Constants

Speed of light, $c \approx 2.998 \times 10^8$ m/s

Magnitude of charge of electron, $e \approx 1.602 \times 10^{-19}$ C

Mass of electron, $m_e \approx 9.109 \times 10^{-31}$ kg

Mass of proton, $m_p \approx 1.673 \times 10^{-27}$ kg

Permitivity of free space, $\epsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{C}^2 \cdot \mathrm{N}^{-1} \cdot \mathrm{m}^{-2}$

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb} \cdot \mathrm{A}^{-1} \cdot \mathrm{m}^{-1}$

Acceleration due to gravity, $g \approx 9.807\ \mathrm{m/s}^{-2}$

B. Solutions to a Quadratic Equation

$$ax^2 + bx + c = 0,$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

C. Derivatives

$$\begin{aligned}
\frac{d}{dx}x^n &= nx^{n-1} \\
\frac{d}{dx}\sin ax &= a\cos ax \\
\frac{d}{dx}\cos ax &= -a\sin ax \\
\frac{d}{dx}e^{ax} &= ae^{ax} \\
\frac{d}{dx}\ln ax &= \frac{a}{x} \\
\frac{d}{dx}\ln f(x) &= \frac{1}{f(x)}\frac{d}{dx}f(x) \\
\frac{d}{dx}\frac{P(x)}{Q(x)} &= \frac{1}{[Q(x)]^2}\left[Q(x)\frac{d}{dx}P(x) - P(x)\frac{d}{dx}Q(x)\right]
\end{aligned}$$

D. Power series

$$
\begin{aligned}
(1+x)^n &= 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots \\
\sin x &= x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots \\
\cos x &= 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots \\
\tan x &= x+\frac{x^3}{3}+\frac{2x^5}{15}+\frac{17x^7}{315}+\cdots \\
e^x &= 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \\
\ln(1+x) &= x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots
\end{aligned}
$$

E. Integrals

$$
\begin{aligned}
\int x^n dx &= \frac{x^{n+1}}{n+1} \quad (n \neq -1) \\
\int \frac{dx}{x} &= \ln x \\
\int \sin ax dx &= -\frac{1}{a}\cos ax \\
\int \cos ax dx &= \frac{1}{a}\sin ax \\
\int e^{ax} dx &= \frac{1}{a}e^{ax} \\
\int \frac{dx}{\sqrt{a^2-x^2}} &= \arcsin\frac{x}{a} \\
\int \frac{x dx}{\sqrt{a^2+x^2}} &= \sqrt{a^2+x^2} \\
\int \frac{dx}{\sqrt{a^2+x^2}} &= \ln(x+\sqrt{a^2+x^2}) \\
\int \frac{dx}{a^2+x^2} &= \frac{1}{a}\arctan\frac{x}{a} \\
\int \frac{dx}{(a^2+x^2)^{3/2}} &= \frac{1}{a^2}\frac{x}{\sqrt{a^2+x^2}} \\
\int \frac{x dx}{(a^2+x^2)^{3/2}} &= -\frac{1}{\sqrt{a^2+x^2}} \\
\int \frac{dx}{\sqrt{(x-a)^2+b^2}} &= \ln\frac{1}{(a-x)+\sqrt{(a-x)^2+b^2}} \\
\int \frac{(x-a)dx}{[(x-a)^2+b^2]^{3/2}} &= -\frac{1}{\sqrt{(x-a)^2+b^2}} \\
\int \frac{dx}{[(x-a)^2+b^2]^{3/2}} &= \frac{x-a}{b^2\sqrt{(x-a)^2+b^2}}
\end{aligned}
$$