NATIONAL UNIVERSITY OF SINGAPORE

PC1143 PHYSICS III

(Semester II: AY 2012-13)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **5 short** questions in Part I and **3 long** questions in Part II. It comprises **5** printed pages.
2. Answer **ALL** the questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The total marks for Part I is 40 and that for Part II is 60.
6. A table of constants and mathematical formulae is attached.

PART I

This part of the examination paper contains **five** short-answer questions on page 2. Answer **ALL** questions.

Question 1
A *spherical capacitor* C has an inner sphere of radius R_1 with a charge of $+Q$ and an outer *concentric* spherical shell of radius R_2 with a charge of $-Q$.

(a) Find the energy density associated with the electric field at any point in the space between the two conductors. [4]

(b) Hence, or otherwise, find the capacitance of C. [4]

Question 2
Of the total energy drawn from a battery of emf E in *charging* a capacitor of capacitance C via a resistor of resistance R, show that half ends up dissipated in the resistor. [8]

Question 3
A long straight wire and a small rectangular wire loop lie in the same plane, Figure 1.

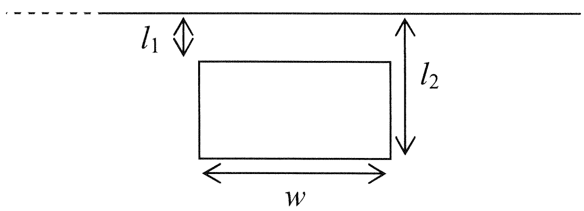

Figure 1

Determine the *mutual inductance* in terms of l_1, l_2, and w. Assume that the wire is very long compared to l_1, l_2, and w, and that the rest of its circuit is very far away compared to l_1, l_2, and w. [8]

Question 4
A series circuit consists of a 100 Ω resistor, a 36 mH inductor, and a 4.0 nF capacitor. The circuit is connected to an ac source with constant voltage amplitude 20 V, and whose frequency can be varied over a wide range.

(a) Find the resonance frequency f_0 of the circuit. [2]

(b) At *resonance*, what is the rms current in the circuit and what are the rms voltages across the inductor and capacitor? [6]

Question 5
The electric field of an electromagnetic wave oscillates in the z-direction and the Poynting vector is given by

$$\mathbf{S}(x,t) = (100\ \mathrm{W/m^2})\sin^2[10x - (3.0\times10^9)t]\,\hat{\mathbf{i}},$$

where x is in meters and t is in seconds.

(a) What is the direction of propagation of the wave? [1]

(b) Find its wavelength and frequency. [2]

(c) Find the electric and magnetic fields. [5]

END OF PART I

PART II

This part of the examination paper contains **three** long questions on pages 3 to 5. Answer **ALL** questions.

Question 6(a)

An electric dipole is *centred* at the origin O, with electric dipole moment $\mathbf{p}$ in the direction of the $+x$-axis (Figure 2).

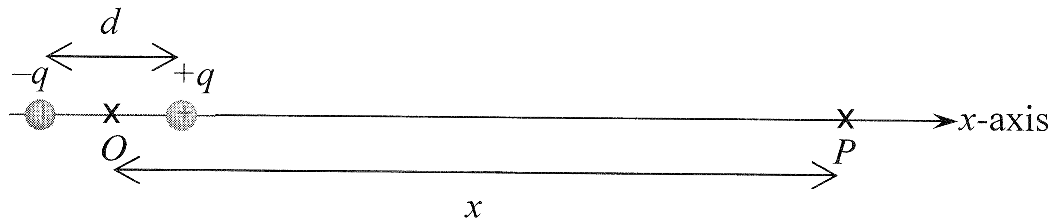

Figure 2

Derive an approximate expression for the electric field at a point P on the x-axis for which x is much larger than d. Express your answer in terms of $\mathbf{p}$. [8]

Question 6(b)

A rod of length $2L$ lies on the x-axis, *centred* on the origin O. It carries a charge per unit length given by

$$\lambda(x) = \lambda_0 \frac{x}{L},$$

where λ_0 is a constant.

(i) Find an expression for the electric potential at all points $x > L$ on the x-axis. [5]

(ii) Hence, find an expression for the electric field at all points $x > L$ on the x-axis. [3]

(iii) Determine the electric dipole moment $\mathbf{p}$ of the rod. Explain very briefly how you obtain your answer. [4]

Question 7(a)
Figure 3 shows a conducting loop lying in the *xy*-plane.

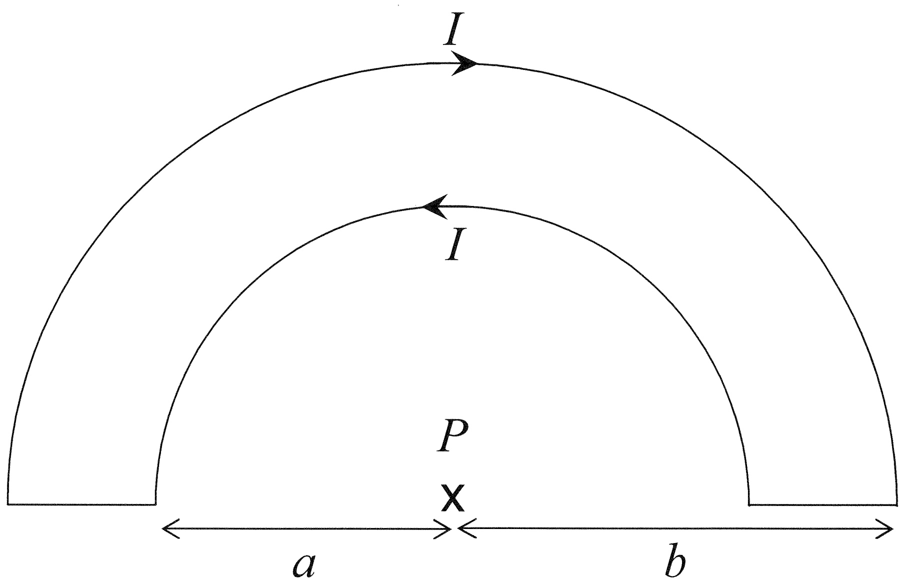

Figure 3

The loop is formed from *concentric* semicircles of radii a and b. If it carries a current I as shown, find the magnetic field at point P, the *common centre*. [8]

Question 7(b)
Two *concentric*, *coplanar* circular current loops have radii a and $2a$. If the magnetic field is zero at their *common centre*, how does the current I_{outer} in the outer loop compare with that I_{inner} in the inner loop? Explain briefly how you arrive at your answer. [4]

Question 7(c)
A solid conducting wire of radius R runs parallel to the z-axis and carries a current density given by

$$\mathbf{J}(r) = J_0\left(1 - \frac{r}{R}\right)\hat{\mathbf{k}},$$

where J_0 is a constant and r the radial distance from the wire axis. Find expressions for
(i) the total current in the wire. [3]
(ii) the magnetic field strength for $r < R$. [3]
(iii)the magnetic field strength for $r > R$. [2]

Question 8(a)

A square conducting loop of side l and resistance R lies in the xy-plane with its sides parallel to the x- and y-axes. It is being moved with a constant velocity $\mathbf{v} = v\,\hat{\mathbf{i}}$. For $x < 0$, there is a uniform magnetic field

$$\mathbf{B} = B_0\hat{\mathbf{k}}\,.$$

For $x > 0$, the field is nonuniform and is given by

$$\mathbf{B} = (B_0 + bx)\,\hat{\mathbf{k}}\,,$$

where B_0 and b are positive constants. At time $t = 0$ the *trailing side* of the loop crosses the y-axis, so the loop is entirely in the nonuniform field region.

(i) Find an expression for the loop current for times $t \geq 0$. [5]

(ii) Which way does the current flow, clockwise or anti-clockwise, as viewed from the positive z-axis? [1]

Question 8(b)

Consider a pair of vertical long conducting rods, a distance l apart. They are connected at the bottom by a resistance R. A conducting bar of mass m runs horizontally between the rods and can slide freely down them while maintaining electrical contact. The whole apparatus is in a uniform magnetic field **B** pointing horizontally and perpendicular to the bar.

(i) Assuming air resistance is negligible, show that if the bar is released from rest at time $t = 0$ its speed at times $t > 0$ is given by $v(t) = v_0[1 - \exp(-\alpha t)]$. [6]

(ii) Hence, or otherwise, find v_0 and α. [2]

Question 8(c)

A conducting disk with radius a, thickness h, and resistivity ρ is inside a solenoid of circular cross section. The disk axis coincides with the solenoid axis. The magnetic field in the solenoid at time t is given by $B = bt$, with b a constant. Find expressions for

(i) the current density in the disk as a function of the distance r from the disk centre. [2]

(ii) the power dissipation in the entire disk. [4]

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END OF PART II

END OF PAPER

A. Fundamental Physical Constants

Speed of light, $c \approx 2.998 \times 10^{8}$ m/s

Magnitude of charge of electron, $e \approx 1.602 \times 10^{-19}$ C

Mass of electron, $m_e \approx 9.109 \times 10^{-31}$ kg

Mass of proton, $m_p \approx 1.673 \times 10^{-27}$ kg

Permittivity of free space, $\epsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{C}^2 \cdot \mathrm{N}^{-1} \cdot \mathrm{m}^{-2}$

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb} \cdot \mathrm{A}^{-1} \cdot \mathrm{m}^{-1}$

Acceleration due to gravity, $g \approx 9.807\ \mathrm{m/s}^{-2}$

B. Solutions to a Quadratic Equation

$$ax^2 + bx + c = 0,$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

C. Derivatives

$$\begin{aligned}
\frac{d}{dx} x^n &= n x^{n-1} \\
\frac{d}{dx} \sin ax &= a \cos ax \\
\frac{d}{dx} \cos ax &= -a \sin ax \\
\frac{d}{dx} e^{ax} &= a e^{ax} \\
\frac{d}{dx} \ln ax &= \frac{a}{x} \\
\frac{d}{dx} \ln f(x) &= \frac{1}{f(x)} \frac{d}{dx} f(x) \\
\frac{d}{dx} \frac{P(x)}{Q(x)} &= \frac{1}{[Q(x)]^2} \left[Q(x) \frac{d}{dx} P(x) - P(x) \frac{d}{dx} Q(x) \right]
\end{aligned}$$

D. Power series

$$
\begin{aligned}
(1+x)^n &= 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \\
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots
\end{aligned}
$$

E. Integrals

$$
\begin{aligned}
\int x^n dx &= \frac{x^{n+1}}{n+1} \ (n \neq -1) \\
\int \frac{dx}{x} &= \ln x \\
\int \sin ax dx &= -\frac{1}{a}\cos ax \\
\int \cos ax dx &= \frac{1}{a}\sin ax \\
\int e^{ax} dx &= \frac{1}{a}e^{ax} \\
\int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin\frac{x}{a} \\
\int \frac{x dx}{\sqrt{a^2 + x^2}} &= \sqrt{a^2 + x^2} \\
\int \frac{dx}{\sqrt{a^2 + x^2}} &= \ln(x + \sqrt{a^2 + x^2}) \\
\int \frac{dx}{a^2 + x^2} &= \frac{1}{a}\arctan\frac{x}{a} \\
\int \frac{dx}{(a^2 + x^2)^{3/2}} &= \frac{1}{a^2}\frac{x}{\sqrt{a^2 + x^2}} \\
\int \frac{x dx}{(a^2 + x^2)^{3/2}} &= -\frac{1}{\sqrt{a^2 + x^2}} \\
\int \frac{dx}{\sqrt{(x-a)^2 + b^2}} &= \ln\frac{1}{(a-x) + \sqrt{(a-x)^2 + b^2}} \\
\int \frac{(x-a)dx}{[(x-a)^2 + b^2]^{3/2}} &= -\frac{1}{\sqrt{(x-a)^2 + b^2}} \\
\int \frac{dx}{[(x-a)^2 + b^2]^{3/2}} &= \frac{x-a}{b^2\sqrt{(x-a)^2 + b^2}}
\end{aligned}
$$