NATIONAL UNIVERSITY OF SINGAPORE

PC1143 PHYSICS III

(Semester II: AY 2013-14)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **5 short** questions in Part I and **3 long** questions in Part II. It comprises **8** printed pages.
2. Answer **ALL** the questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. Only non-programmable calculators are allowed.
6. The total marks for Part I is 40 and that for Part II is 60.
7. A table of constants and mathematical formulae is attached.

PART I

This part of the examination paper contains **five** short-answer questions on pages 2 to 3. Answer **ALL** questions.

1. Consider an **isolated** spherical conductor S of radius R carrying a net charge Q.
   (a) Calculate the total work W needed to assemble this charge Q by bringing infinitesimal charges dq from infinity and depositing them on the surface of S. [2]
   (b) Calculate the electrostatic energy U_E stored in the electric field **E** outside the spherical conductor. [2]
   (c) Is U_E less than, equal to, or greater than W? Explain very briefly the **physical significance** of your answer. [2]
   (d) Hence, or otherwise, find the *capacitance* C of the spherical conductor. [2]

2. A toroidal coil of square cross section has **inner radius** R and side l as shown in Figure 1. The coil consists of N turns, and carries a current I.

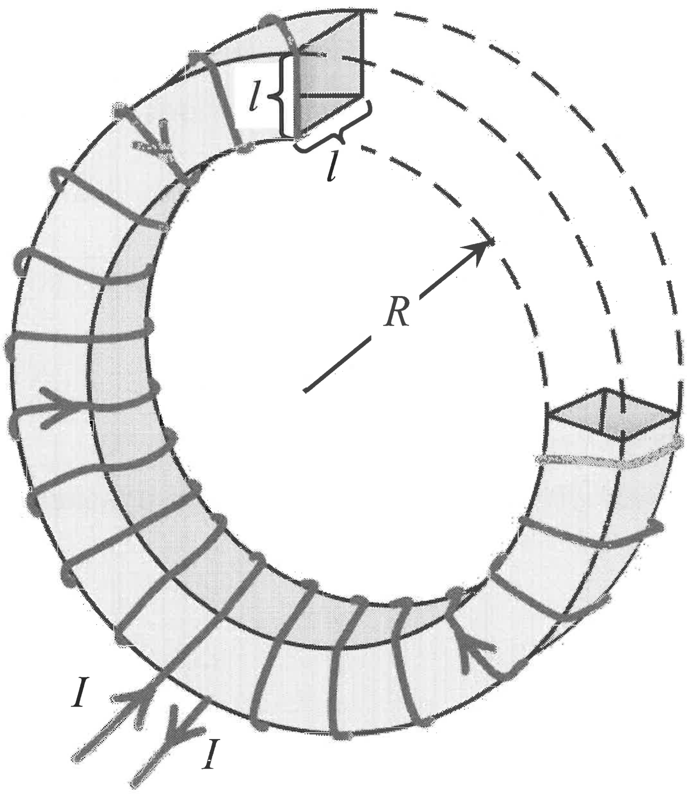

Figure 1

Do NOT assume that magnetic field is uniform across a cross section.

(a) What is the total magnetic energy stored in the toroid? [4]
(b) Hence, or otherwise, determine the *inductance* L of the toroid. [2]
(c) Show that L reduces to the inductance of a **long solenoid** when $R >> l$. [2]

3. The switch in Figure 2 has been open for a long time. It is closed at time $t = 0\,\text{s}$.

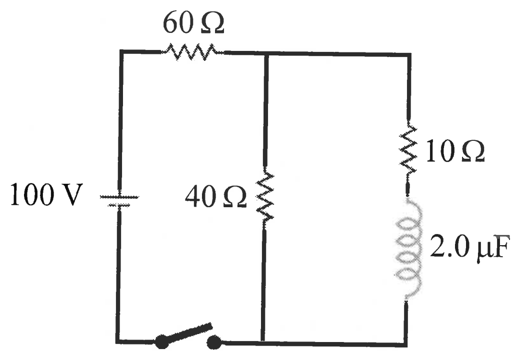

Figure 2

(a) What is the current in the $40\,\Omega$ resistor immediately after the switch is closed? [2]
(b) Find an expression for the current I through the inductor as a function of time t. [4]
(c) What is the current in the $10\,\Omega$ resistor after the switch has been closed for a long time? [2]

4. Consider the series *RLC* circuit in Figure 3.

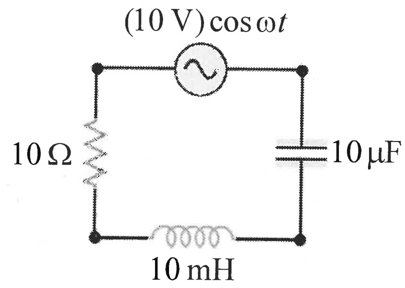

Figure 3

(a) Find the *impedance* of the circuit. Express your answer in terms of ω. [2]
(b) What is the *resonance frequency*, in both rad/s and Hz? [2]
(c) Find V_R and V_L at resonance. [2]
(d) How can V_L be larger than 10 V? Explain. [2]

5. At one instant, the electric field **E** and magnetic field **B** at one point of an *electromagnetic wave* are

$$\mathbf{E} = (200\hat{\mathbf{i}} + 300\hat{\mathbf{j}} - 50\hat{\mathbf{k}})\ \text{V/m},$$

and

$$\mathbf{B} = B_0(7.30\hat{\mathbf{i}} - 7.30\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}})\ \mu\text{T}.$$

(a) What are the values of α and B_0? [4]
(b) What is the *Poynting vector* at this time and position? [4]

End of Part I

PART II

This part of the examination paper contains **three** long questions on pages 4 to 8. Answer **ALL** questions.

6. A pair of **equal** but **opposite** point charges, $+q$ and $-q$, lies on the x axis at $x=-a$ and $x=+a$ respectively, as shown in Figure 4.

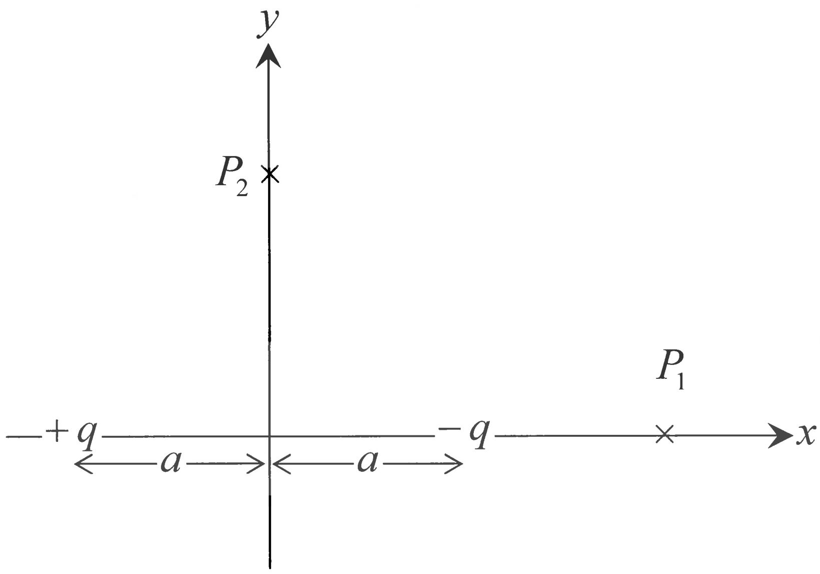

Figure 4

(a) Find the electric potential V_1 at point $P_1(x,0)$ on the x axis. [2]

(b) Write down the mathematical relationship between the electrostatic field **E** and potential V at a point in space. [2]

(c) Hence, find the electric field $\mathbf{E}_1$ at point P_1. [2]

(d) Find the electric field $\mathbf{E}_2$ at point $P_2(0,y)$ on the y axis. [4]

(e) What is the electric potential V_2 at P_2? Does your answer contradict that in (d)? Explain briefly. [4]

(f) Find $\mathbf{E}_1$ where $x >> a$, and $\mathbf{E}_2$ where $y >> a$. Express your answers in terms of the **electric dipole moment** of the system of two charges. What is common to your answers? [6]

7. A straight section of wire of length L carries a current I, as shown in Figure 5.

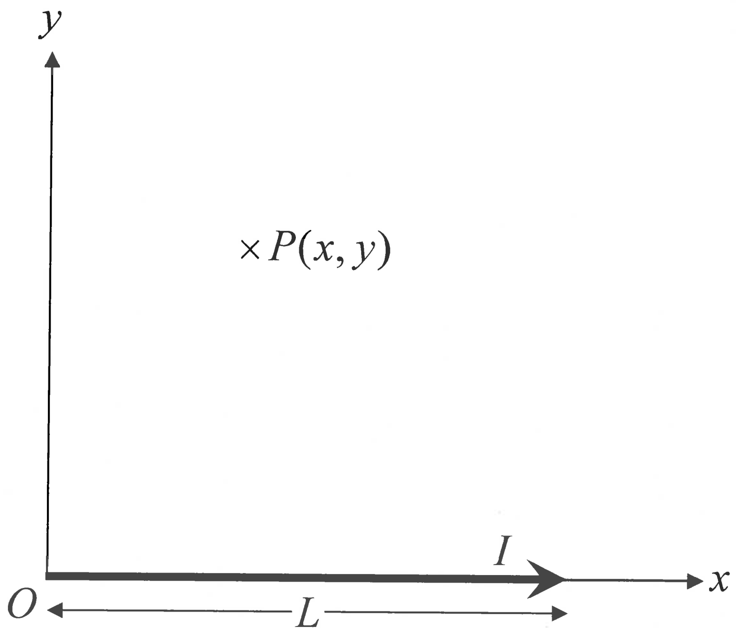

Figure 5

(a) Show that the magnetic field **B** associated with this segment at point P is given by

$$\mathbf{B} = \frac{\mu_0 I}{4\pi y}\left[\frac{x}{\sqrt{x^2+y^2}} - \frac{x-L}{\sqrt{(x-L)^2+y^2}}\right]\hat{\mathbf{k}}.$$

[6]

(b) Hence, find **B** when P is very close to the current-carrying wire. Explain how you could apply Ampere's law to determine **B** in this case. [4]

(c) Find the magnetic field **B** at point Q in Figure 6. [6]

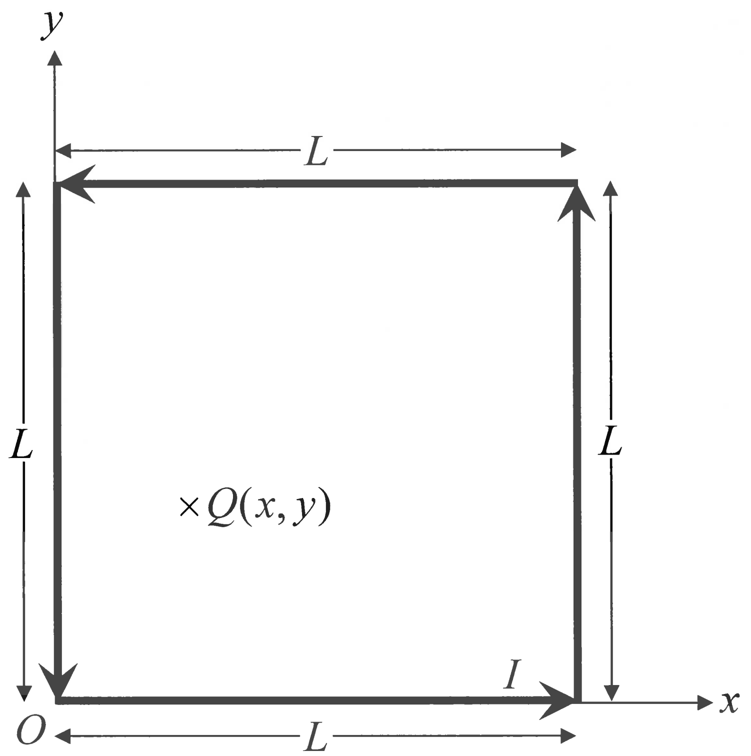

Figure 6

(d) Is the magnetic field **B** at R greater or less than at the centre of the square? Justify your answer. [4]

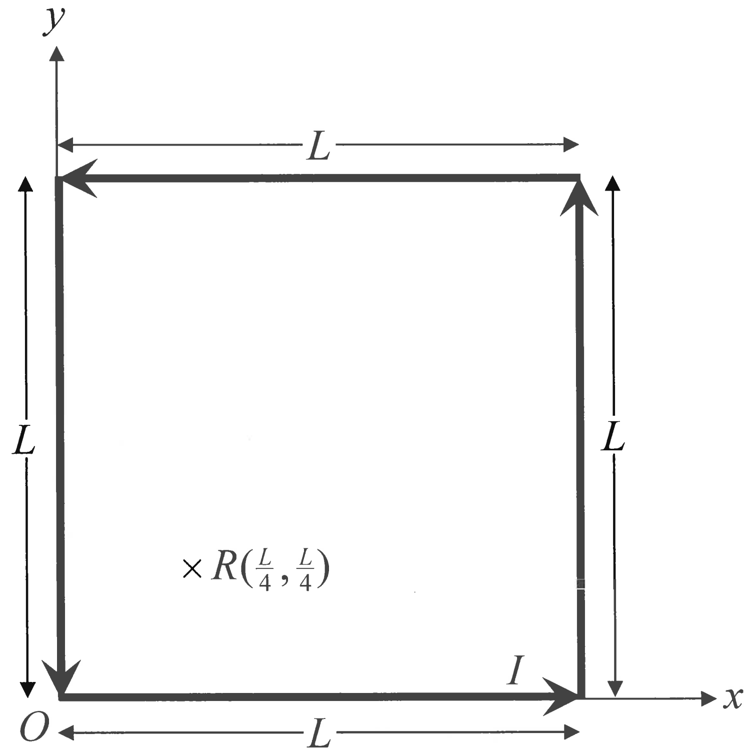

Figure 7

8. Consider a conducting ring with radius R, uniform circular cross-sectional area A, and resistivity ρ, lying with its plane perpendicular to a uniform magnetic field B as shown in Figure 8.

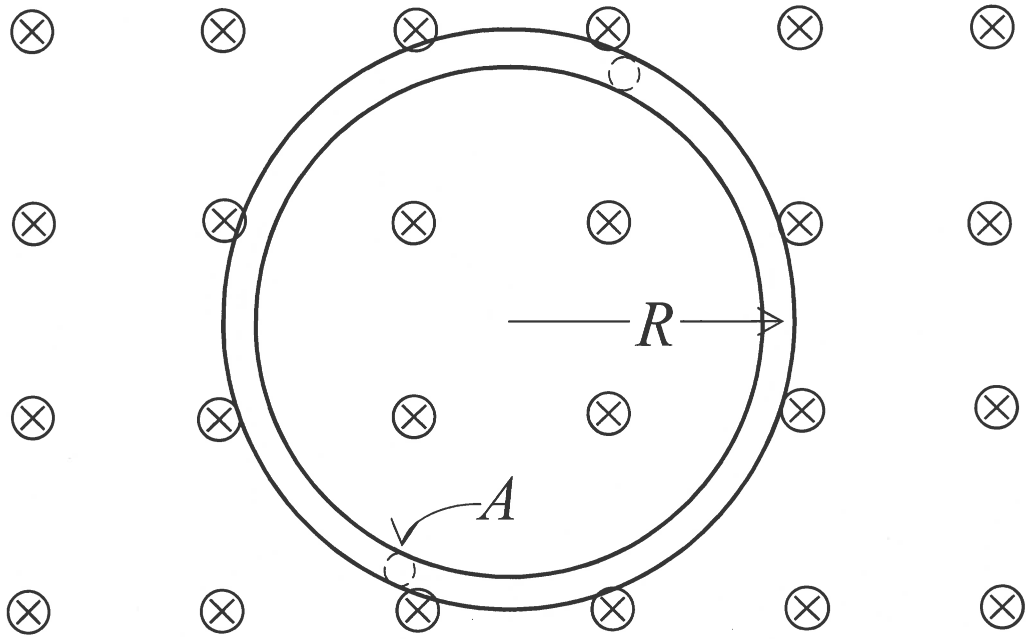

Figure 8

(a) Suppose the magnetic field B at time t is given by

$$B = B_0[1 - \exp(-bt)],$$

with b and B_0 positive constants.

(i) Find an expression for the induced current density in the ring as a function of time t. [6]

(ii) Hence, find by integrating the induced current over time the total charge that moves around the ring as B increases from zero to B_0. [4]

(b) Figure 9 shows a generator consisting of a conducting rod of length R that rotates with angular speed ω about a central axis through O while making contact with the conducting ring.

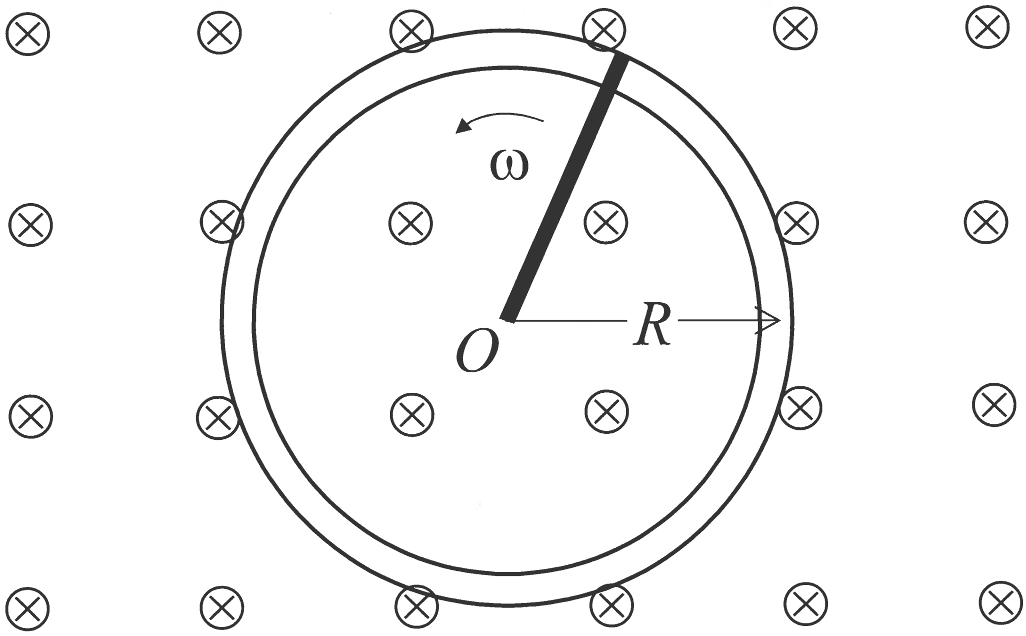

Figure 9

Suppose the magnetic field $B = B_0$ is now constant.

(i) Find an expression for the electric field in the rod as a function of the distance r from the central axis, when the conduction electrons in the rod are in equilibrium. [2]

(ii) Hence, or otherwise, find an expression for the emf induced in this generator. Specify if the emf induced is directed away or towards the central axis. [3]

(iii) Now, wires from the axis and ring carry power to a load. If the induced current in the circuit is I_0, find an expression for the rate of work done by an external agent to maintain the angular speed of the rod at ω. [3]

(iv) Hence, or otherwise, find an expression for the load resistance. [2]

YY

End of Part II

– End of Paper –

A. Fundamental Physical Constants

Speed of light, $c \approx 2.998 \times 10^8$ m/s

Magnitude of charge of electron, $e \approx 1.602 \times 10^{-19}$ C

Mass of electron, $m_e \approx 9.109 \times 10^{-31}$ kg

Mass of proton, $m_p \approx 1.673 \times 10^{-27}$ kg

Permittivity of free space, $\epsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{C^2 \cdot N^{-1} \cdot m^{-2}}$

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb \cdot A^{-1} \cdot m^{-1}}$

Acceleration due to gravity, $g \approx 9.807\ \mathrm{m/s^{-2}}$

B. Solutions to a Quadratic Equation

$$ax^2 + bx + c = 0,$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

C. Derivatives

$$\begin{aligned}
\frac{d}{dx}x^n &= nx^{n-1} \\
\frac{d}{dx}\sin ax &= a\cos ax \\
\frac{d}{dx}\cos ax &= -a\sin ax \\
\frac{d}{dx}e^{ax} &= ae^{ax} \\
\frac{d}{dx}\ln ax &= \frac{a}{x} \\
\frac{d}{dx}\ln f(x) &= \frac{1}{f(x)}\frac{d}{dx}f(x) \\
\frac{d}{dx}\frac{P(x)}{Q(x)} &= \frac{1}{[Q(x)]^2}\left[Q(x)\frac{d}{dx}P(x) - P(x)\frac{d}{dx}Q(x)\right]
\end{aligned}$$

D. Power series

$$
\begin{aligned}
(1+x)^n &= 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \\
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots
\end{aligned}
$$

E. Integrals

$$
\begin{aligned}
\int x^n dx &= \frac{x^{n+1}}{n+1} \ (n \neq -1) \\
\int \frac{dx}{x} &= \ln x \\
\int \sin ax dx &= -\frac{1}{a} \cos ax \\
\int \cos ax dx &= \frac{1}{a} \sin ax \\
\int e^{ax} dx &= \frac{1}{a} e^{ax} \\
\int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin \frac{x}{a} \\
\int \frac{x dx}{\sqrt{a^2 + x^2}} &= \sqrt{a^2 + x^2} \\
\int \frac{dx}{\sqrt{a^2 + x^2}} &= \ln(x + \sqrt{a^2 + x^2}) \\
\int \frac{dx}{a^2 + x^2} &= \frac{1}{a} \arctan \frac{x}{a} \\
\int \frac{dx}{(a^2 + x^2)^{3/2}} &= \frac{1}{a^2} \frac{x}{\sqrt{a^2 + x^2}} \\
\int \frac{x dx}{(a^2 + x^2)^{3/2}} &= -\frac{1}{\sqrt{a^2 + x^2}} \\
\int \frac{dx}{\sqrt{(x-a)^2 + b^2}} &= \ln \frac{1}{(a-x) + \sqrt{(a-x)^2 + b^2}} \\
\int \frac{(x-a)dx}{[(x-a)^2 + b^2]^{3/2}} &= -\frac{1}{\sqrt{(x-a)^2 + b^2}} \\
\int \frac{dx}{[(x-a)^2 + b^2]^{3/2}} &= \frac{x-a}{b^2\sqrt{(x-a)^2 + b^2}}
\end{aligned}
$$