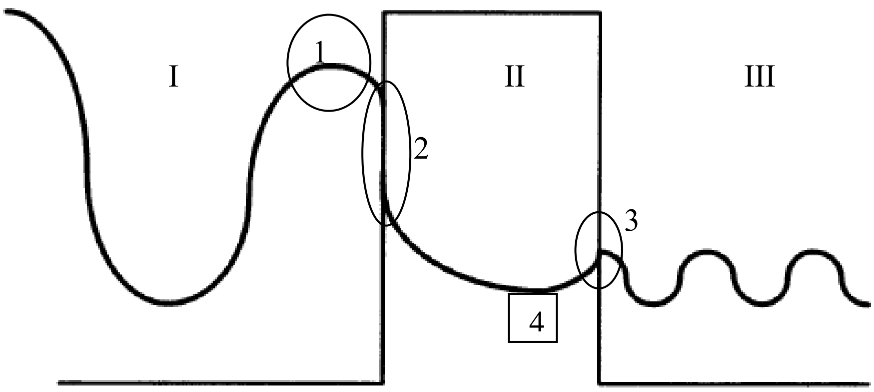
Solution to PC1144 0506 Sem 2 Paper

Part I

1



Mistakes:

(1) The wavefunction is sinuisoidal in the region before the barrier, so it should not decrease at the peak.

(2) The wavefunction is discontinuous, the correct wavefunction should be continuous, the wavefunction in region I and wavefunction in region II at the barrier has the same value.

(3) The wavefunction is discontinuous, although the value is the same, but the first derivative of the wavefunction in region II and region III is not the same. A continuous wavefunction means that the wavefunctions at the boundary have the same value and the first derivative are equal too.

(4) The wavefunction in region II is an exponential decay. The wavefunction drawn is not an exponential decay curve.

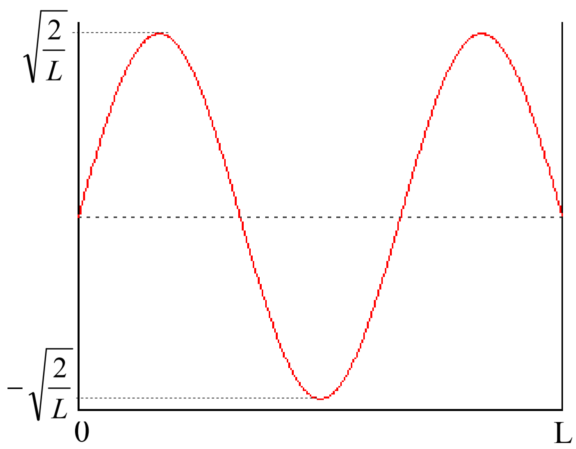
2 Using $KE = \frac{1}{2}mv^2 = \frac{k_e e^2}{2r}$

$$v = \sqrt{\frac{k_e e^2}{mr}} = \sqrt{\frac{(8.988\times10^{9})(1.6\times10^{-19})^2}{(9.11\times10^{-31})(0.0529\times10^{-9})}} = 2.19\times10^{6}\,ms^{-1}$$

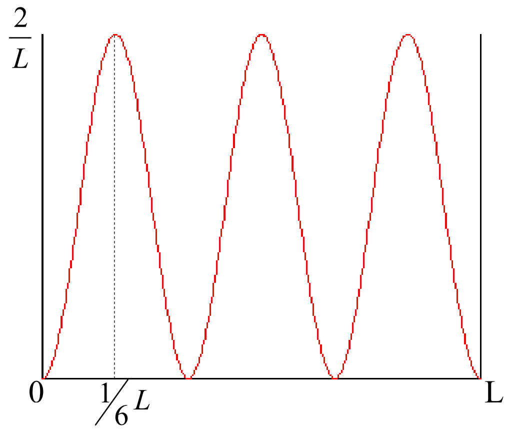
$$v = r\omega = r2\pi f$$

Number of revolutions $= \frac{v}{2\pi r} = \frac{2.19\times10^{6}}{2\pi(0.0529\times10^{-9})} = 6.57\times10^{15}\,revolutions$

3 Wavefunction:



Probability density profile:



Probability of the particle being found between $x = 0$ and $x = 1/6$ L is $\frac{1}{6}$

The total probability of the probability density profile is 1, since at 1/6 L, the area under graph is 1/6, so the probability is 1/6.

4 Heisenberg's Uncertainty Principle: $\Delta x \Delta p \geq \frac{1}{2}\hbar$

$$p = \frac{h}{\lambda} \Rightarrow \Delta p = \frac{h}{\Delta \lambda}$$

$$\Delta x \frac{h}{\Delta \lambda} \geq \frac{1}{2}\hbar$$

$$\Delta x \frac{1}{\Delta \lambda} \geq \frac{1}{4\pi}$$

$$\Delta x \geq \frac{\Delta \lambda}{4\pi} = \frac{(1\times10^{-6})(300\times10^{-9})}{4\pi} = 2.39\times10^{-14} m$$

$$\Delta x \geq 2.39\times10^{-14} m$$

5(i) All conserved. Reaction can proceed.

5(ii) All conserved. Reaction can proceed.

5(iii) All conserved except lepton number. Reaction cannot proceed.

5(iv) All conserved except strangeness. Reaction cannot proceed.

Part II

6(a) $\Delta x' = \gamma(\Delta x - v\Delta t)$ $\qquad$ $\Delta t' = \gamma(\Delta t - v\Delta x / c^2)$

$$\frac{\Delta x'}{\Delta t'} = \frac{\Delta x - v\Delta t}{\Delta t - v\Delta x / c^2} \Rightarrow \frac{dx'}{dt'} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

Since $u'_x = \frac{dx'}{dt'}$, $u_x = \frac{dx}{dt}$, we have $u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$ (shown)

6(b) For non-relativistic case, $\frac{v}{c^2} \approx 0$

$$u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} = u_x - v$$ (Galilean velocity transformation) (shown)

6(c) $u_x = c$

$$u'_x = \frac{c - v}{1 - \frac{vc}{c^2}} = \frac{c - v}{\frac{c - v}{c}} = c$$ (shown)

6(d) Using inverse Lorentz transformation, $u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}}$

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}} = \frac{0.9c + 0.5c}{1 + \frac{(0.9c)(0.5c)}{c^2}} = 0.966c$$

Velocity of positron in the laboratory frame = $0.966c$

6(e) Velocity of gamma ray = c

Since gamma ray is an electromagnetic radiation which travels in speed of light, by Einstein's postulate, light travels in speed of light in any inertial frame, so velocity of gamma ray is speed of light.

7(a) $E = \gamma mc^2$ $\quad\quad p = \gamma mu$

$E^2 = \gamma^2 (mc^2)^2$ $\quad\quad p^2 = \gamma^2 m^2 u^2$

$$\gamma^2 = \frac{1}{1 - (u/c)^2} = \frac{c^2}{c^2 - u^2} = \frac{E^2}{(mc^2)^2}$$

$$\gamma^2 u^2 = (\gamma^2 - 1)c^2$$

$$p^2 = (\gamma^2 - 1)m^2c^2 = (\frac{E^2}{(mc^2)^2} - 1)m^2c^2$$

$$p^2 m^2 c^4 = (E^2 - (mc^2)^2)m^2c^2$$

$E^2 = p^2c^2 + (mc^2)^2$ (shown)

7(b) If the total energy of a particle is much higher than its rest mass energy,

$$E^2 - (mc^2)^2 \approx E^2 = p^2c^2$$

$$p^2 = \frac{E^2}{c^2}$$

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

Wavelength of particle is similar to that of a photon of the same energy. (shown)

7(c) Wavelength of gamma ray $=\dfrac{hc}{E}=\dfrac{(6.626\times10^{-34})(3.0\times10^{8})}{5\times1.6\times10^{-13}}=2.48\times10^{-13}m$

$$p_{electron}=\sqrt{\frac{E^2-(mc^2)^2}{c^2}}=\sqrt{\frac{(8.0\times10^{-13})^2-(9.11\times10^{-31}\times9.0\times10^{16})^2}{9.0\times10^{16}}}=2.65\times10^{-21}kgms^{-1}$$

$$\lambda_{electron}=\frac{h}{p_{electron}}=\frac{6.626\times10^{-34}}{2.65\times10^{-21}}=2.50\times10^{-13}m$$

$$\text{\% difference}=\frac{2.50\times10^{-13}-2.48\times10^{-13}}{2.48\times10^{-13}}\times100\%=0.529\%$$

7(d) Total energy of proton = $8.0\times10^{-13}+(1.67\times10^{-27})(3.0\times10^{8})^2=1.511\times10^{-10}J$

$$p_{proton}=\sqrt{\frac{E^2-(mc^2)^2}{c^2}}=\sqrt{\frac{(1.511\times10^{-10})^2-(1.67\times10^{-27}\times9.0\times10^{16})^2}{9.0\times10^{16}}}=5.18\times10^{-20}kgms^{-1}$$

$$\lambda_{proton}=\frac{h}{p_{proton}}=\frac{6.626\times10^{-34}}{5.18\times10^{-20}}=1.28\times10^{-14}m$$

$$\text{ratio}=\frac{1.28\times10^{-14}}{2.48\times10^{-13}}=0.0515$$

8(a) The energy of the system undergoing radioactive decay must conserved. In beta decay experiment, it is found that beta particles from a single type of nucleus are emitted over a continuous range of energies. The kinetic energy of the system after the decay is equal to the decrease in mass-energy of the system, but this is not found in beta decay as beta particles are emitted over a continuous range of energies. As such, Pauli proposed a third particle to carry away the "missing" energy and momentum, which is named neutrino by Fermi.

8(b) Any living organisms in Earth will have a fixed ratio of ^{14}C to ^{12}C. When the organism dies, it no longer exchanges carbon dioxide from the atmosphere, and therefore the ^{14}C in the organism starts to decay according to the radioactive decay laws. By measuring the current activity of the ^{14}C in a dead organism, using the activity equation, we can know how long ago the organism died.

8(c) Using $R=R_0e^{-\lambda t}$, $t_{1/2}=5730 years$

$$4=32e^{-\frac{0.693}{5730}t}$$

$$\frac{0.693}{5730}t=\ln 8$$

$$t=\frac{5730\ln 8}{0.693}=17200 years$$

8(d) The calculated age of the skeleton will be smaller, this is because the presence of radioactive thorium will increase the count rate measured by the GM counter. The increase in count rate will mean that the measured count rate is close to the initial count rate, resulting a smaller calculated age.