PC1144

NATIONAL UNIVERSITY OF SINGAPORE

PC1144 PHYSICS IV

(Semester 2: April 2006)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **8 questions** and comprises **6** printed pages.
2. Answer **all five** questions in Part I and any **two out of three** questions in Part II.
3. This is a CLOSED BOOK examination.
4. The total mark for part I is 40 and that for part II is 60.

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PART I

This part of the examination paper contains **five** (5) short-answer questions (page 2 – page 4). **Answer ALL questions.**

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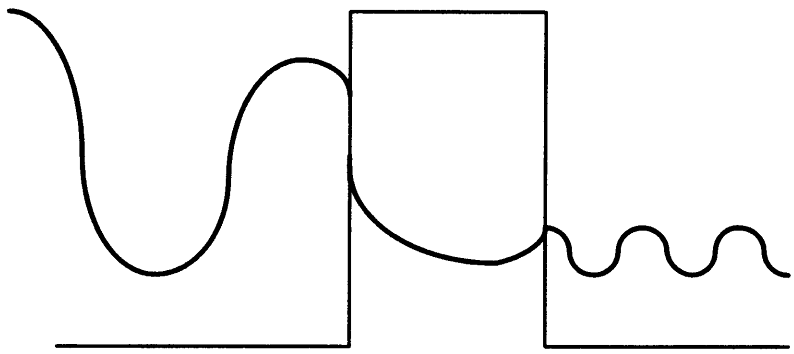
1) A particle travelling from left to right impinges on a rectangular potential barrier. A student has drawn a sketch of the wavefunction before, inside and after the barrier.



Unfortunately the student has made a series of mistakes in the drawing. Identify these mistakes, and **explain why** they are errors.

2) In a hydrogen atom, an electron in an excited state will quickly drop into a lower energy state with the release of a photon. If the lifetime of the 1^{st} excited state in the hydrogen atom is 10^{-8} seconds, use the Bohr theory to calculate how many revolutions around the nucleus an electron in the 1^{st} excited state will make before it drops into the ground state.

[**Hint:** The Bohr radius (a_0) is 0.0529 nm, and the kinetic energy of an electron in a Bohr orbit is $KE = k_e e^2/2r$ where r is the radius of the orbit. k_e is the Coulomb constant = 8.988×10^9 N m^2 C^{-2}]

3) A particle in a rectangular well of infinite sides can be described by the wavefunction

$\psi(x) = \sqrt{(2/L)}\ \sin(3\pi x / L)$

Draw the wavefunction and the probability density profile of this particle, and estimate the probability of the particle being found in the box between x = 0, and x = 1/6 L, where L is the width of the box. Explain your answer.

4) What is the uncertainty in the position of a photon of wavelength 300nm, if its wavelength can be measured to one part per million?

5) Conservation laws are important in Particle Physics to identify both reaction and decay mechanisms. Using the conservation of charge, lepton number, baryon number and strangeness, deduce whether or not the following reactions can proceed. Explain your answers.

(i) $p + p \rightarrow K^{+} + p + \Lambda^{0}$

(ii) $\pi^{-} + p \rightarrow K^{0} + \Sigma^{0}$

(iii) $n \rightarrow p + e^{-} + \upsilon_{e}$

(iv) $\pi^{-} + p \rightarrow \pi^{0} + \Lambda^{0}$

[Hint: If you wish, you can use the table of particle properties on the next page]

Category	Particle Name	Symbol	Anti-particle	Rest Mass (MeV/c^2)	B	L_e	L_μ	L_τ	S	Lifetime (s)	Principal Decay Modes[a]
Photon	Photon	γ	Self	0	0	0	0	0	0	Stable	
Leptons	Electron	e^-	e^+	0.511	0	+1	0	0	0	Stable	
	Neutrino (e)	ν_e	$\bar{\nu}_e$	0(?)	0	+1	0	0	0	Stable	
	Muon	μ^-	μ^+	105.7	0	0	+1	0	0	2.20×10^{-6}	$e^-\bar{\nu}_e\nu_\mu$
	Neutrino (μ)	ν_μ	$\bar{\nu}_\mu$	0(?)	0	0	+1	0	0	Stable	
	Tau	τ^-	τ^+	1784	0	0	0	−1	0	$<4 \times 10^{-13}$	$\mu^-\bar{\nu}_\mu\nu_\tau$, $e^-\bar{\nu}_e\nu_\tau$, hadrons
	Neutrino (τ)	ν_τ	$\bar{\nu}_\tau$	0(?)	0	0	0	−1	0	Stable	
Hadrons											
Mesons	Pion	π^+	π^-	139.6	0	0	0	0	0	2.60×10^{-8}	$\mu^+\nu_\mu$
		π^0	Self	135.0	0	0	0	0	0	0.83×10^{-16}	2γ
	Kaon	K^+	K^-	493.7	0	0	0	0	+1	1.24×10^{-8}	$\mu^+\nu_\mu$, $\pi^+\pi^0$
		K_S^0	$\overline{K_S^0}$	497.7	0	0	0	0	+1	0.89×10^{-10}	$\pi^+\pi^-$, $2\pi^0$
		K_L^0	$\overline{K_L^0}$	497.7	0	0	0	0	+1	5.2×10^{-8}	$\pi^\pm e^\mp(\bar{\nu})_e$, $\pi^\pm\mu^\mp(\bar{\nu})_\mu$, $3\pi^0$
	Eta	η^0	Self	548.8	0	0	0	0	0	$<10^{-18}$	2γ, 3π
Baryons	Proton	p	$\bar{p}$	938.3	+1	0	0	0	0	Stable	
	Neutron	n	$\bar{n}$	939.6	+1	0	0	0	0	920	$pe^-\bar{\nu}_e$
	Lambda	Λ^0	$\overline{\Lambda^0}$	1115.6	+1	0	0	0	−1	2.6×10^{-10}	$p\pi^-$, $n\pi^0$
	Sigma	Σ^+	$\overline{\Sigma}^-$	1189.4	+1	0	0	0	−1	0.80×10^{-10}	$p\pi^0$, $n\pi^+$
		Σ^0	$\overline{\Sigma}^0$	1192.5	+1	0	0	0	−1	6×10^{-20}	$\Lambda^0\gamma$
		Σ^-	$\overline{\Sigma}^+$	1197.3	+1	0	0	0	−1	1.5×10^{-10}	$n\pi^-$
	Xi	Ξ^0	$\overline{\Xi^0}$	1315	+1	0	0	0	−2	2.9×10^{-10}	$\Lambda^0\pi^0$
		Ξ^-	Ξ^+	1321	+1	0	0	0	−2	1.64×10^{-10}	$\Lambda^0\pi^-$
	Omega	Ω^-	Ω^+	1672	+1	0	0	0	−3	0.82×10^{-10}	$\Xi^0\pi^0$, Λ^0K^-

[a] A notation in this column such as $p\pi^-$, $n\pi^0$ means two possible decay modes. In this case, the two possible decays are $\Lambda^0 \rightarrow p + \pi^-$ or $\Lambda^0 \rightarrow n + \pi^0$.

Table of particle properties

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PART II

This part of the examination paper contains THREE (3) long-answer questions from page 5 to 6. **Answer any TWO questions.**

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6 a) The Lorentz transformation equations describing the co-ordinate transformation from a stationary reference frame S to a reference frame $S^{/}$ in the x-direction are

$\Delta x^{/} = \gamma(\Delta x - v\,\Delta t)$ and

$\Delta t^{/} = \gamma(\Delta t - v\,\Delta x / c^2)$

where v is the relative velocity between the two frames.

Show that the velocity transformation is given by $\mathbf{u_x}^{/} = (\mathbf{u_x} - \mathbf{v}) / [1 - (\mathbf{v}/\mathbf{c}^2)\mathbf{u_x}]$

What is the corresponding velocity transformation from frame $S^{/} \rightarrow S$?

b) Show that in the non-relativistic case, the expression for the velocity transformation corresponds to the Galilean velocity transformation.

c) A photon travels at the speed of light c. Prove that any observer in a different inertial frame also sees the photon travelling at the same speed c.

d) A radioactive nucleus is accelerated in a linear particle accelerator, and reaches a speed of 0.5c as measured in the laboratory frame of reference. The nucleus then undergoes β-decay and emits a positron with a speed of 0.9c with respect to the nucleus, along the same direction of motion. What is the velocity of the positron in the laboratory frame?

e) A radioactive nucleus is accelerated in a linear particle accelerator, and reaches a speed of 0.5c as measured in the laboratory frame of reference. The nucleus then undergoes γ-decay at right angles to the direction of motion. What is the velocity of the gamma ray in the laboratory frame? Explain your answer.

7) a) A particle has a total energy $E = \gamma mc^2$ and momentum $p = \gamma mu$. Derive the relationship $E^2 = p^2c^2 + (mc^2)^2$

b) Show that if the total energy of a particle is much higher than its rest mass energy, then the wavelength of this particle is similar to that of a photon of the same energy.

c) Calculate the **% difference** between the wavelength of an **electron** with **total energy** of 5 MeV and the wavelength of a 5 MeV gamma ray.

d) Calculate the **ratio** between the wavelength of a **proton** with a **kinetic energy** of 5 MeV and the wavelength of a 5 MeV gamma ray.

8) a) Describe briefly how the neutrino was predicted long before it was discovered.

b) Carbon 14 beta decays with a half life of 5730 years. Describe briefly how this can be used as a dating technique.

c) A female skeleton is found in an old cave. Around the skeleton's neck is a gold necklace which has attached to it a leather holder containing a large jewel. The jewel is removed from the leather holder, and the holder measured to have a beta activity of 4 counts per minute. A similar piece of modern leather with the same shape and mass is found to have a beta activity of 32 counts per minute. Approximately how long ago did the woman die?

[Hint: The activity equation is $R = R_0 e^{-\lambda t}$ where $\lambda = 0.693 / T_{1/2}$]

d) On closer inspection in the laboratory, it is observed that the jewel contains an abnormally high level of radioactive thorium. How would this affect the true age of the skeleton, and why?

END OF PAPER: FW