

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$


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1)a)  $v = 0.9990c$

$l = 4600m$

In the frame of the scientist,

$$t = \frac{4600}{0.9990c} = 1.55 \times 10^{-5} s$$

$$t' = \frac{t}{\gamma}$$

$$\frac{1}{\gamma} = \sqrt{(1 - 0.999^2)}$$

$$t' = \sqrt{(1 - 0.999^2)} \frac{4600}{0.9990c}$$

$$= 6.93 \times 10^{-7} s = 0.693 \mu s$$


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b)  $v = 0.9990c$

$t' = 0.693 \mu s$

$L = 2.056 \mu m$

$L' = 205.6 m$

2)

(a)  $E_k = 10^{13} MeV$

$$\gamma mc^2 = 10^{13+6} \times 1.6 \times 10^{-19}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{mc^2}{1.6}$$

$$1 - \frac{v^2}{c^2} = 8.5 \times 10^{-21}$$

$$\frac{v^2}{c^2} \approx 1$$

$v = 0.999999c$

$L = 10^6 LY.$

$t' \approx t = 10^6 Y$

(b)  $L = \gamma L'$

$$L' = \frac{L}{\gamma}$$

$$= \frac{10^6 LY.}{1.08 \times 10^{10}}$$

$$= 9.26 \times 10^{-5} LY.$$

3)

$$\psi(x) = \sqrt{\frac{2}{L}} \sin(n\pi \frac{x}{L})$$

$$\begin{aligned} \psi^2(x) &= \frac{2}{L} \sin^2(n\pi \frac{x}{L}) \\ &= \frac{2}{L} \frac{1}{2} \left[ 1 - \cos(2n\pi \frac{x}{L}) \right] \end{aligned}$$

$$\begin{aligned} P(0.45L < x < 0.55L) &= \int_{0.45L}^{0.55L} \frac{1}{L} \left[ 1 - \cos(2n\pi \frac{x}{L}) \right] dx \\ &= \frac{1}{L} \left[ x - \frac{L}{2n\pi} \sin(2n\pi \frac{x}{L}) \right]_{0.45L}^{0.55L} \\ &= \left[ 0.55 - \frac{1}{2n\pi} \sin(2n\pi 0.55) \right] - \left[ 0.45 - \frac{1}{2n\pi} \sin(2n\pi 0.45) \right] \\ &= \left[ 0.1 + \frac{1}{2n\pi} (0.309 + 0.309) \right] \end{aligned}$$

Since  $n = 1$

$$\begin{aligned} P(0.45L < x < 0.55L) &= \left[ 0.1 + \frac{1}{2\pi} (0.309 + 0.309) \right] \\ &= 0.198 = 19.8\% \end{aligned}$$

Classically, there would be equal probabilities for all positions of the particle, which means the probability of  $0.45L < x < 0.55L$  will be :

$$P(0.45L < x < 0.55L) = \frac{(0.55 - 0.45)L}{L} = \frac{0.1L}{L} = 0.1 = 10\%$$

The probability of particle in  $0.45L$  to  $0.55L$  is lower classically by calculation.

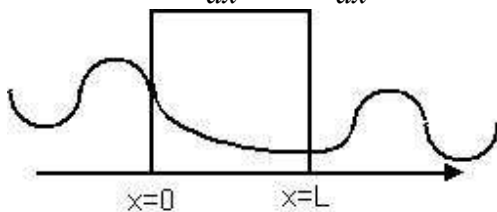
4)

$$\psi_1(0) = \psi_2(0)$$

$$\psi_2(L) = \psi_3(L)$$

$$\text{At } x = 0, \frac{d\psi_1}{dx} = \frac{d\psi_2}{dx}$$

$$\text{At } x = L, \frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$$



b) By the Schrodinger equation,

Region 1

$$\frac{d\psi_1}{dx} = \frac{-2m}{\hbar^2} E \psi_1$$

The wavefunction is a sinusoidal function.

Region 2

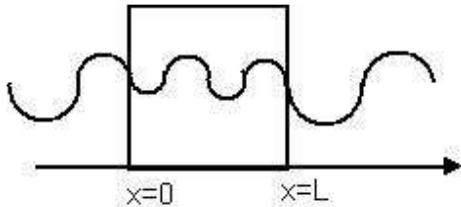
$$\frac{d\psi_1}{dx} = \frac{-2m}{\hbar^2} (E - U) \psi_1$$

$$(E - U) > 0$$

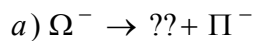
The wavefunction is a sinusoidal function upon solution.

Region 3

The wavefunction is similar to that of the one in region 1 since there is only kinetic energy experienced by particle. E remains unchanged.



5)



Conservation of charge,

$$-1 = z - 1$$

$$z = 0$$

Conservation of Baryon number,

$$1 = B + 0$$

$$B = +1$$

Conservation of strangeness,

(conservation does not hold since this is weak interaction)

$$S = -2, -1$$

Conservation of Lepton number,

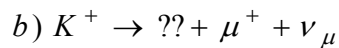
$$0 = L + 0$$

$$L = 0$$

$$L = 0; B = +1; z = 0; S = -2, -1$$

The possible particles

$$\Lambda^0, \Sigma^0, \Xi^0$$



Conservation of charge,

$$+1 = z + 1 + 0$$

$$z = 0$$

Conservation of Baryon number,

$$0 = B + 0 + 0$$

$$B = 0$$

Conservation of Strangeness,

(conservation does not hold since this is weak interaction)

$$S = +1, +3, -1$$

Conservation of Lepton,

$$0 = L - 1 + 1$$

$$L = 0$$

$$L = 0; B = 0; z = 0; S = +1, +3, -1$$

The possible particles

$$\Pi^0, \eta^0$$

6) a)

$$h \frac{c}{\lambda} = h \frac{c}{\lambda_0} + E_k$$

$E_k$  : maximum kinetic energy of  
photoelectron

i) Electron can only absorb 1 photon according to quantum physics. If  $h \frac{c}{\lambda} < h \frac{c}{\lambda_0}$ , then electron would not have enough energy to escape the surface since its energy is below the work function. Classically, electron can absorb wavelike light slowly and eventually escape once it has enough energy.

$$\text{ii) } h \frac{c}{\lambda} = h \frac{c}{\lambda_0} + E_k = hf$$

Energy photons are  $E=hf$ , which are independent of intensity but proportional to frequency of photons. Classically, light in waveform will also have greater energy at greater intensity. Thus, if light is made out of photons, their will be greater energy only if frequency increases.

iii)  $f = \frac{c}{\lambda}$ ,  $E = hf$ , but as  $\lambda$  increase,  $f$  decreases, thus energy of photon will be less.

Energy absorbed by electron will be less.

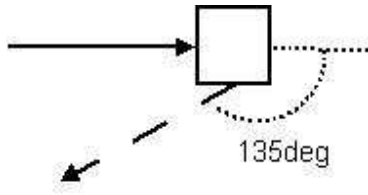
iv) Classically, electrons absorb wavelike light gradually before escaping,  $Pt=E$ . Thus,  $t \gg 0s$ , but modern theory suggests that electron absorbs photon and gains energy instantly.

$$\text{b) } i) \phi_0 = h \frac{c}{\lambda_0} = 1.88eV$$

$$\text{ii) } h \frac{c}{\lambda} = h \frac{c}{\lambda_0} + E_k$$

$$E_k = hc \left( \frac{1}{\lambda} + \frac{1}{\lambda_0} \right) \\ = 4.27eV$$

$$h \frac{c}{\lambda} = 6.1498eV$$



$$c) \lambda - \lambda_0 = \frac{h}{mc} (1 - \cos \theta)$$

$$\lambda_0 = 0.2 \text{ nm}$$

$$\lambda - \lambda_0 = \frac{h}{mc} (1 - \cos 135^\circ)$$

$$= 4.18 \times 10^{-12}$$

$$\lambda_2 = 0.204 \text{ nm}$$

$$\lambda_1 = 0.200 \text{ nm}$$

Some photons were not scattered, thus possess the original energy. The ones scattered will share some energy with the electrons, thus will possess less energy and greater wavelength.

7)

a)i)

$$mvr = n\hbar$$

$$E_k = \frac{ke^2}{2r}$$

$$\frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

$$v = \frac{n}{mr2\pi}$$

$$v^2 = \frac{n^2 h^2}{m^2 r^2 (2\pi)^2}$$

$$\frac{n^2 h^2}{mr(2\pi)^2} = ke^2$$

$$r = \frac{n^2 h^2}{m(2\pi)^2 ke^2}$$

ii)

$$E = \frac{-ke^2}{2r}$$

$$a_0 = \frac{h^2}{(2\pi)^2 mke^2}$$

$$r_n = a_0 n^2$$

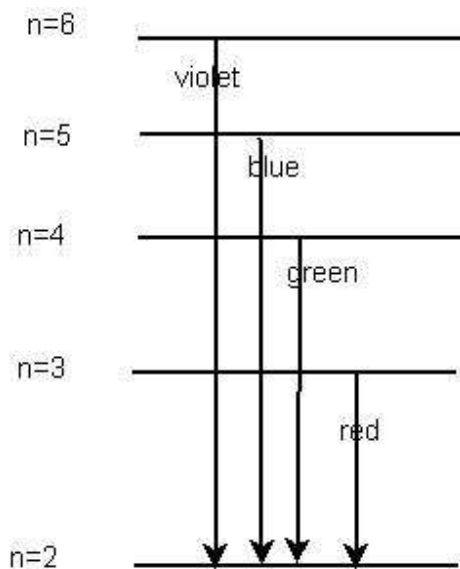
$$E = \frac{-ke^2}{2a_0 n^2}$$

$$E = \frac{-ke^2}{2a_0n^2}$$

$$\begin{aligned} \text{(iii)} \quad &= -\left[\frac{1}{n^2}\right] \frac{ke^2}{2a_0} \\ &= 2.17 \times 10^{-18} J \\ &= 13.6 eV \end{aligned}$$

b)

i) Spectral lines appear when excited electrons drop into lower energy levels, where the energy difference is converted into photons at different frequencies.



$$\frac{hc}{\lambda} = \frac{ke^2}{2a_0} \left( \frac{1}{n_i^2} - \frac{1}{n^2} \right)$$

$$\frac{1}{\lambda} = R \left( \frac{1}{n_i^2} - \frac{1}{n^2} \right)$$

$$= R \left( \frac{1}{4} - \frac{1}{n^2} \right)$$

$$\frac{1}{n^2} = \left( \frac{1}{4} - \frac{1}{R\lambda} \right)$$

$$\text{(red)} \rightarrow \frac{1}{n_r^2} = 0.11115$$

$$\text{(green)} \rightarrow \frac{1}{n_g^2} = 0.06253$$

$$\text{(blue)} \rightarrow \frac{1}{n_b^2} = 0.04007$$

$$\text{(violet)} \rightarrow \frac{1}{n_v^2} = 0.027848$$

$$n_v = 6$$

$$n_b = 5$$

$$n_g = 4$$

$$n_r = 3$$

$$E = h \frac{c}{\lambda}$$

$$= 1.613 \times 10^{-18} \text{ J}$$

ii)

$$\text{Since } 1.613 \times 10^{-18} = \frac{hc}{\lambda} = \frac{ke^2}{2a_0} \left( 1 - \frac{1}{4} \right)$$

The electron is dropping from level 2 to level 1.

iii) As potential difference is smaller, the electrons of the atomic gas do not have enough energy to be excited to  $n=4,5$  or 6. The only way for emission to occur is for electrons to fall from  $n=2$  to  $n=1$ , thus only the 122nm line is observed.

8)

i) The difference of binding energy between daughter and parent nucleus during the fission process will be the energy harnessed by the reactor.

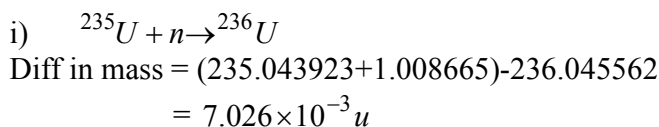
ii) When a parent nucleus encounter the fission process, the neutrons released by it could be used to (once slowed down) to start another fission process. This keeps the nuclear reaction going to generate power continuously.

iii) The reproduction constant will determine whether the chain reaction will proceed, it is the average number of nucleon generated per fission event. A self sustained chain reaction will happen when  $k=1$ . For  $k<1$ , reaction will die out. For  $k>1$ , run-away reaction will occur.

iv) The moderator is used to slow down neutrons to enable them to be absorbed by fissionable nuclei in order to continue the chain reaction.

v) Neutrons are absorbed by control rods at a certain rate to control the rate of chain reaction (by controlling the neutron flux).

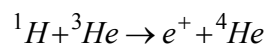
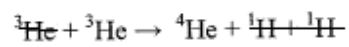
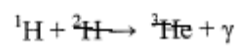
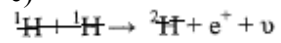
b)



$$\begin{aligned}
 E &= mc^2 \\
 &= (7.026 \times 10^{-3})u \times c^2 \\
 &= 1.029 \times 10^{-12} J \\
 &= 6.43 \text{ MeV}
 \end{aligned}$$

ii) This is the energy that would be released when excited uranium becomes stable in the fission process.

c)



$$E = 4(938.3) \text{ MeV} - (0.511 + 3727.4) = 25.289 \text{ MeV}$$

ii) Much greater compared to the fission process.