NATIONAL UNIVERSITY OF SINGAPORE

PC1144 - Physics IV Modern Physics

(Semester II: AY 2007-08)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **8 questions** and comprises 7 printed pages.
2. Answer **all five** questions in Part I and **all three** questions in Part II.
3. This is a CLOSED BOOK examination.
4. The total mark for Part I is 40 and that for Part II is 60.

PC1144 — PHYSICS 4
PART I

This part of the examination paper contains **five** (5) short-answer questions (page 2 – page 4). **Answer ALL questions.**

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1) In relativity:

   (i) What is meant by the concept of total energy?

   (ii) Show that $E^2 - p^2c^2 = (mc^2)^2$

   (iii) Using the expression derived in (ii), show that the de Broglie wavelength of a particle is approximately the same as that of a photon, providing the total energy of the particle is much greater than its rest mass.

2) An atom in a material is assumed to oscillate with simple harmonic motion in one dimension.

   i) Sketch the *wavefunctions* and *probability densities* corresponding to the first 3 energy states of the oscillating atom (n = 0, 1, 2).

   ii) Describe qualitatively any differences between the quantum mechanical and the classical scenarios corresponding to the *second* energy state (n = 1).

3) Using the Heisenberg uncertainty principle, calculate the minimum energy (in MeV) of a neutron when confined to a nucleus of diameter 10^{-14} m.

4) (i) What is meant by the binding energy of a nucleus?

(ii) Calculate, using the table below, the nuclear binding energy of carbon 12 (6 protons and 6 neutrons) and express your result in MeV.

	Mass		
Particle	**kg**	**u**	**MeV/c^2**
Proton	$1.672\ 62 \times 10^{-27}$	1.007 276	938.28
Neutron	$1.674\ 93 \times 10^{-27}$	1.008 665	939.57
Electron	$9.109\ 39 \times 10^{-31}$	$5.485\ 79 \times 10^{-4}$	0.510 999
^1_1H atom	$1.673\ 53 \times 10^{-27}$	1.007 825	938.783
^4_2He	$6.644\ 66 \times 10^{-27}$	4.001 506	3727.38
$^{12}_6C$ atom	$1.992\ 65 \times 10^{-26}$	12	11 177.9

5) A K^+ particle with kinetic energy 150 MeV, decays into two π^+ particles and one π^- particle. Range measurements of the energies of the decay products using photographic emulsion show that the two π^+ particles have kinetic energies of 68.6 and 80.8 MeV respectively, and that the π^- particle has a kinetic energy of 75.5 MeV. Calculate the mass of the K^+.

[Use the table on page 4 where required].

Table of Particle properties:

Category	Particle Name	Symbol	Anti-particle	Rest Mass (MeV/c^2)	B	L_e	L_μ	L_τ	S	Lifetime (s)
Photon	Photon	γ	Self	0	0	0	0	0	0	Stable
Leptons	Electron	e^-	e^+	0.511	0	+1	0	0	0	Stable
	Neutrino (e)	ν_e	$\bar{\nu}_e$	0(?)	0	+1	0	0	0	Stable
	Muon	μ^-	μ^+	105.7	0	0	+1	0	0	2.20×10^{-6}
	Neutrino (μ)	ν_μ	$\bar{\nu}_\mu$	0(?)	0	0	+1	0	0	Stable
	Tau	τ^-	τ^+	1784	0	0	0	−1	0	$< 4 \times 10^{-13}$
	Neutrino (τ)	ν_τ	$\bar{\nu}_\tau$	0(?)	0	0	0	−1	0	Stable
Hadrons Mesons	Pion	π^+	π^-	139.6	0	0	0	0	0	2.60×10^{-8}
		π^0	Self	135.0	0	0	0	0	0	0.83×10^{-16}
	Kaon	K^+	K^-		0	0	0	0	+1	1.24×10^{-8}
		K^0_S	$\overline{K^0_S}$	???	0	0	0	0	+1	0.89×10^{-10}
		K^0_L	$\overline{K^0_L}$		0	0	0	0	+1	5.2×10^{-8}
	Eta	η^0	Self	548.8	0	0	0	0	0	$< 10^{-18}$
Baryons	Proton	p	$\bar{p}$	938.3	+1	0	0	0	0	Stable
	Neutron	n	$\bar{n}$	939.6	+1	0	0	0	0	920
	Lambda	Λ^0	$\overline{\Lambda^0}$	1115.6	+1	0	0	0	−1	2.6×10^{-10}
	Sigma	Σ^+	$\overline{\Sigma}^-$	1189.4	+1	0	0	0	−1	0.80×10^{-10}
		Σ^0	$\overline{\Sigma}^0$	1192.5	+1	0	0	0	−1	6×10^{-20}
		Σ^-	$\overline{\Sigma}^+$	1197.3	+1	0	0	0	−1	1.5×10^{-10}
	Xi	Ξ^0	$\overline{\Xi^0}$	1315	+1	0	0	0	−2	2.9×10^{-10}
		Ξ^-	Ξ^+	1321	+1	0	0	0	−2	1.64×10^{-10}
	Omega	Ω^-	Ω^+	1672	+1	0	0	0	−3	0.82×10^{-10}

PART II

This part of the examination paper contains THREE (3) long-answer questions from page 5 to 7. **Answer ALL questions.**

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6 a) The Lorentz transformation equations describing the co-ordinate transformation from a stationary reference frame S to a reference frame $S^{/}$ are

$$\Delta x^{/} = \gamma(\Delta x - v\,\Delta t)$$

and $$\Delta t^{/} = \gamma(\Delta t - v\,\Delta x / c^2)$$

where v is the relative velocity between the two frames.

If an object is observed in the S frame to have a speed u_x, derive an expression for $u_x^{/}$, the relative velocity of the object in the $S^{/}$ frame.

b) An enemy spaceship races away from Earth at a speed of 0.7c, as observed by an observer on Earth. The same observer on Earth sees a defender spaceship giving chase at a speed of 0.75c. What relative speed does the defender spaceship captain believe he is overtaking the enemy ship?

c) While the defender spaceship is chasing the enemy ship, the enemy ship launches a photon torpedo in a reverse direction directly towards the defender ship. What speed does the photon torpedo strike the defender spaceship? Can the defender spaceship captain take evasive action and avoid the photon torpedo? Explain your answer.

7) a) Consider a particle of mass m, trapped in a rectangular box with **infinite** walls. Assume the particle is represented by the wavefunction **y (x) = A sin (n π x / L),** where A is a constant, L is the width of the box and n is an integer number.

(i) Derive an expression for the energy levels of the allowed states for the particle.

(ii) Show that the energy difference between the ground state and the second excited state is $\mathbf{h^2/mL^2}$

b) An electron is trapped in a rectangular box of **infinite** sides, and is in the **first excited state**. If the width of the box is 0.50 nm, what is the wavelength of the photon that is emitted when the electron drops into the ground state?

c) Consider an electron trapped in a box with **finite** walls. Draw schematic diagrams showing i) the wavefunctions and ii) probability densities, for the three lowest energy states of the electron. How does this differ from that of an electron trapped in a box of infinite sides?

8) a) Discuss briefly (using diagrams where necessary) the physical mechanisms underlying alpha decay, beta decay and gamma decay.

b) Carbon 14 beta decays with a half life of 5730 years. Describe briefly how this can be used as a dating technique.

c) A male skeleton is found in an ancient ceremonial site deep in the jungle by a team of scientists. A ceremonial axe with a carved wooden handle is found buried in the skull of the skeleton. The axe handle is measured to have a ^{14}C beta count of 26 counts per minute. A piece of wood of the same shape and mass cut from a nearby live tree measures 31 counts per minute. Approximately how long ago did the person die? Why is it especially important in this case to estimate the errors in the measurements?

[continued on the next page]

d) One of the scientists who carried out the dating measurements described in 3c above is also interested in astronomy. He remembered that over the last few years there had been much more surface activity on the sun, and that on average a higher number of solar flares from the sun's surface had been observed. He mentions to his fellow scientists that this will affect the dating of the axe handle. Why, and in what way?

END OF PAPER: FW