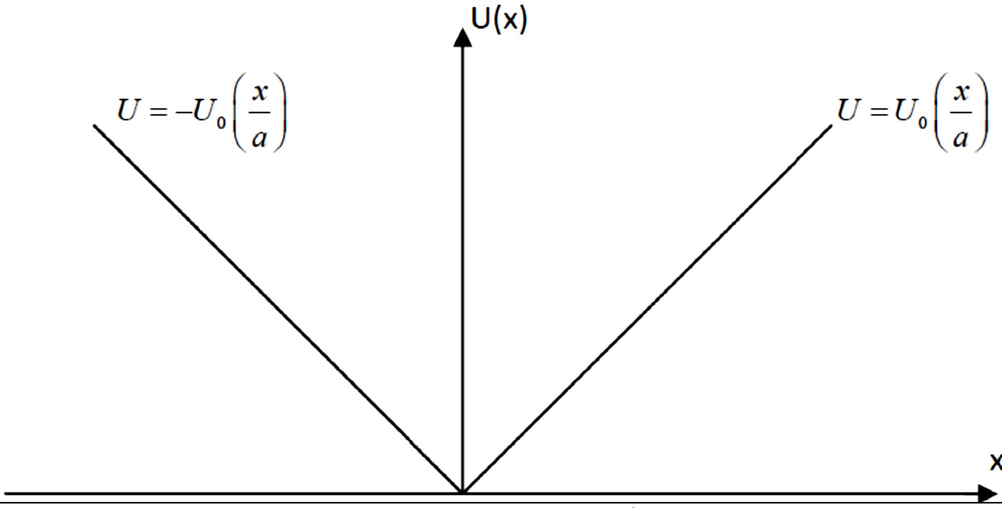
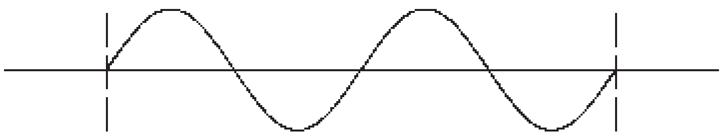
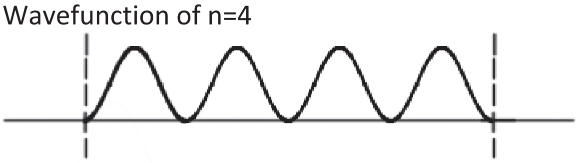
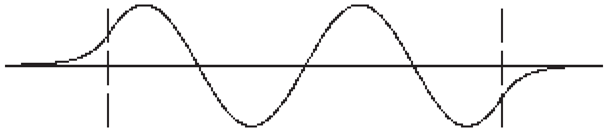
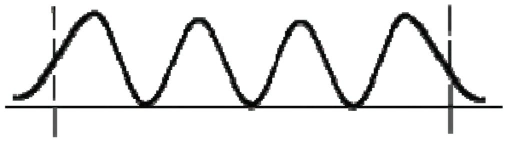
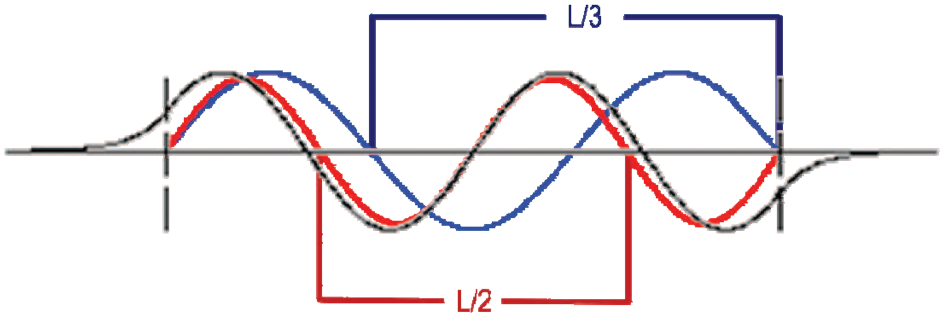
1(a)	$$E_{kinetic} = \frac{p^2}{2m_e} = eV \quad \Rightarrow \quad p = \sqrt{2m_e eV}$$ $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}} = 2.74 \times 10^{-10}\mathrm{m}$$
1(b) & 1(c)	Intensity, $$I = \frac{P}{A} = \frac{neV}{At} \propto \frac{n}{t}$$ $$I_1 = 100 = \frac{1}{2}A_1^2, \qquad A_1 = 14.14$$ $$I_2 = 900 = \frac{1}{2}A_2^2, \qquad A_2 = 42.43$$ The superposition of the 2 electron beams, induce the final intensity: $$x(t) = x_1(t) + x_2(t) = A_1\cos\,(\omega_1 t) + A_2\cos\,(\omega_2 t + \phi)$$ $$I = \overline{x^2(t)} = \frac{1}{2}A_1^2 + \frac{1}{2}A_2^2 + 2A_1A_2\,\overline{\cos\,(\omega_1 t)\cos\,(\omega_2 t + \phi)}$$ It is easy to see that $\overline{\cos(\omega_1 t)\cos\,(\omega_2 t + \phi)} = 0$ under either of these two conditions: • $\omega_1 \neq \omega_2$ • ϕ is randomly changing with time That means, if the waves are coherent, it is not 0 and the intensity would instead be: $$I = \frac{1}{2}A_1^2 + \frac{1}{2}A_2^2 + A_1A_2\cos(\phi)$$ $$\max\,(I) = \frac{1}{2}(A_1 + A_2)^2, \qquad \min\,(I) = \frac{1}{2}(A_1 - A_2)^2$$ So intensities do not necessarily add up. The answer should now be clear, given the assumption of coherence: $$I_{center} = \frac{1}{2}(A_1 + A_2)^2 = 1600 \text{ electrons/s}$$ $$I_{dark} = \frac{1}{2}(A_1 - A_2)^2 = 400 \text{ electrons/s}$$
2(a)	$$\lambda_f - \lambda_i = \frac{h}{mc}(1 - \cos\theta)$$ where λ_f is the wavelength after collision, and λ_i is the wavelength before collision. The longer wavelength is due to photons deflected due to collision with the nuclei of the atoms. Since nucleus is much more massive than electrons, photons colliding with the nuclei will lose little amount of energy. The momentum and hence wavelength of these photons remains approximately the same. The shorter wavelength is due ot photons deflected due to collision with the electrons. During collision, these photons will lose a significant fraction of their energy to the electron, so as to obey the conservation of momentum. Therefore, the deflected photons will have less energy which means less momentum and longer wavelength. Recall $E = pc = hc/\lambda$.
2(b)	$$\lambda_f - \lambda_i = \frac{h}{mc}(1 - \cos\theta), \qquad \lambda = \frac{h}{p}, \qquad E = pc \Rightarrow p = \frac{E}{c}$$ $$\frac{hc}{E_f} - \frac{hc}{E_i} = \frac{h}{mc}(1 - \cos\theta) \quad \Rightarrow \quad \frac{mc^2}{E_f} - \frac{mc^2}{E_i} = 1 - \cos\theta$$

	since $E_i \gg mc^2$, $\frac{mc^2}{E} \approx 0 \Rightarrow \frac{mc^2}{E_f} \approx 1 - \cos\theta$ As $\theta = 180°$, $\frac{mc^2}{E_f} = 2$, $\therefore E_f - \frac{1}{2}mc^2 = 0.25\text{MeV}$
3(a)	
3(b)	$E = U + K = U + \frac{p^2}{2m}$ Suppose that we are to determine the state of the particle within the range from 0 to x, so as to make the math easier and that we do not have to take care of the minus sign on the left half of $U(x)$. Hence: $\Delta x = x$. Besides, let us estimate Δp as p. Therefore: $p \geq \hbar/x$. Hence: $E(x) \geq U(x) + \frac{\hbar^2}{2mx^2}, \quad E_{min}(x) = U(x) + \frac{\hbar^2}{2mx^2}$ Now we need to find the "most likely" x. Stability implies minimum energy: $\left.\frac{dE_{min}}{dx}\right\|_{x=x_{stable}} = 0$ $x_{stable} = \left(\frac{a\hbar^2}{mU_o}\right)^{1/3}, \quad \Rightarrow \quad E_{min} = \frac{U_o}{a}x_{stable} + \frac{\hbar^2}{2mx_{stable}^2}$
4(a)	Conservation of charge, conservation of L_e, L_μ and L_τ.
4(b)	$r = \frac{h^2 n^2 \varepsilon_0}{\pi m e^2} \Rightarrow r \propto \frac{1}{m}$ $m_{reduced} = \frac{m_\mu m_p}{m_\mu + m_p} = 94.998\text{MeV}$ $r = \frac{m_e}{m_{reduced}} a_0 = 2.85 \times 10^{-13}\text{m}$

Solutions provided by: **NUS Physics Society** (Q1a, Q2, Q3a, Q4-Q8), **Nguyen Phan Minh** (Q1b, Q1c, Q3b)

5(a)	$E_\pi = m_\pi c^2$ $E_\mu = \sqrt{(p_\mu c)^2 + (m_\mu c^2)^2}$ $E_{\bar{v}_\mu} = p_{\bar{v}_\mu} c$ By conservation of energy: $E_\pi = E_\mu + E_{\bar{v}_\mu}$ $\Rightarrow m_\pi c^2 = \sqrt{(p_\mu c)^2 + (m_\mu c^2)^2} + p_{\bar{v}_\mu} c$ by conservation of linear momentum: $p_\mu = -p_{\bar{v}_\mu}$ $\Rightarrow m_\pi c^2 = \sqrt{(p_{\bar{v}_\mu} c)^2 + (m_\mu c^2)^2} + p_{\bar{v}_\mu} c$ $\Rightarrow (m_\pi c^2 - p_{\bar{v}_\mu} c)^2 = (p_{\bar{v}_\mu} c)^2 + (m_\mu c^2)^2$ $\Rightarrow (m_\pi c^2)^2 - 2(m_\pi c^2)(p_{\bar{v}_\mu} c) = (m_\mu c^2)^2$ $\Rightarrow 2(m_\pi c^2) E_{\bar{v}_\mu} = \left\|(m_\mu c^2)^2 - (m_\pi c^2)^2\right\|$ $\Rightarrow E_{\bar{v}_\mu} = \left\|\frac{(m_\mu c^2)^2 - (m_\pi c^2)^2}{2(m_\pi c^2)}\right\| = \left\|\frac{105.7^2 - 139.6^2}{2\times 139.6}\right\| = 29.8\text{MeV}$
5(b)	$E_{\bar{v}_\mu} = p_\mu c$ $E_\mu = \gamma m_\mu c^2 = \sqrt{(p_\mu c)^2 + (m_\mu c^2)^2}$ $\Rightarrow \gamma^2 = \frac{(p_\mu c)^2 + (m_\mu c^2)^2}{(m_\mu c^2)^2} = 1 + \frac{(p_\mu c)^2}{(m_\mu c^2)^2}$ $\Rightarrow \gamma = \sqrt{1 + \left(\frac{29.8}{105.7}\right)^2} = 1.038982$ $\Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.038982$ $\Rightarrow 1 - \frac{v^2}{c^2} = \left(\frac{1}{1.038982}\right)^2$ $\Rightarrow v = c\sqrt{1-\left(\frac{1}{1.038982}\right)^2} = 0.27135c = 8.14\times 10^7\,\text{ms}^{-1}$
6(a)(i)	Refer to lecture notes
6(a)(ii)	using $t' = \frac{t - ux/c^2}{\sqrt{1-u^2/c^2}}$ let $t_1' = \frac{t_1 - ux/c^2}{\sqrt{1-u^2/c^2}}$ and $t_2' = \frac{t_2 - ux/c^2}{\sqrt{1-u^2/c^2}}$ $\Delta t' = t_1' - t_2' = \frac{t_1 - ux/c^2}{\sqrt{1-u^2/c^2}} - \frac{t_2 - ux/c^2}{\sqrt{1-u^2/c^2}} = \frac{\Delta t}{\sqrt{1-u^2/c^2}} = \gamma \cdot \Delta t$ $\Delta t' = \gamma \cdot \Delta t$, where both events occur at the same position in S frame. Δt is the proper time

	using $x=\dfrac{x'+ut'}{\sqrt{1-u^2/c^2}}$ let $x_1=\dfrac{x_1'+ut_1'}{\sqrt{1-u^2/c^2}}$ and $x_2=\dfrac{x_2'+ut_2'}{\sqrt{1-u^2/c^2}}$ $\Delta l_0=x_1-x_2=\dfrac{(x_1'-x_2')+u(t_1'-t_2')}{\sqrt{1-u^2/c^2}}$ since $t_1'=t_2'$, $\Delta l_0=\gamma\cdot\Delta l$ $\Delta l_0=\dfrac{\Delta l}{\gamma}$,where Δl_0 is the proper length
6(b)(i)	$u=0.6c \qquad \gamma=1.25$ using $\Delta l=\dfrac{\Delta l_0}{\gamma}$, length of ship viewed by observer on satellite, $\Delta l=\dfrac{30}{1.25}=24\text{m}$
6(b)(ii)	$t=\dfrac{\Delta l}{u}=\dfrac{24}{0.6c}=\dfrac{40}{c}$ s
6(b)(iii)	$t_1'=\dfrac{\Delta l_0}{u}=\dfrac{30}{0.6c}=\dfrac{50}{c}$ s
6(b)(iv)	using $t=\dfrac{t'+ux'/c^2}{\sqrt{1-u^2/c^2}}$ $t_1=1.25\cdot\left[\dfrac{30}{c}+\dfrac{(0.6c)}{c^2}\left(\dfrac{30}{c}\times 0.6c-30\right)\right]=\dfrac{28.5}{c}$ s
7(a)(i)	 Wavefunction of n=4  Probability Density Function of n=4
7(a)(ii)	$\psi=\sqrt{\dfrac{2}{L}}\sin\dfrac{4\pi x}{L}$
7(a)(iii)	$k=\dfrac{2\pi}{\lambda}=\dfrac{4\pi}{L}\Rightarrow\lambda=\dfrac{L}{2}$ $p=\dfrac{h}{\lambda}$ $E=\dfrac{p^2}{2m}=\dfrac{h^2}{2m\lambda^2}=\dfrac{2h^2}{mL^2}$

7(b)(i)	 Wavefunction  Probability Density Function
7(b)(ii)	 The red wavefunction is the n=4 state of electron in infinite box and blue wavefunction is the n=3 state of electron in box with infinite U_0. The black wavefunction is n=4 state of electron in box with infinite U_0. Clearly, the wavelength of 3rd excited state of electron in box with finite U_0 has wavelength slightly larger than L/2 but slightly smaller than L/3.
7(b)(iii)	$E_n = \frac{h^2 n^2}{8mL^2}$ $E_3 = \frac{9h^2}{8mL^2}$ $\quad E_4 = \frac{2h^2}{mL^2}$ therefore, the energy is between E_3 and E_4.
7(b)(iv)	$E_n = \frac{h^2 n^2}{8mL^2}$ for $n = 9,\ E_9 = \frac{81h^2}{8mL^2}$ for $n = 10,\ E_9 = \frac{100h^2}{8mL^2}$ The energy of each state for the electron in the box with finite U_0 is lower than that of box with infinite U_0. Therefore, the n=9 state will have energy lesser than E_9 above, which is less than $10h^2/(mL^2)$. Therefore, the allowed bound states will include n=9, with a total of 9 allowed bound states.
8(a)	C_1A: strong force attraction between nucleon, which is proportional to the number of nucleons (A). $-C_2A^{2/3}$: the nucleons on the outer surface are less tightly bound by strong forces, this is to make correction in the strong force attraction due to surface nucleons. This is proportional to the surface area $4\pi r^2$. Since $A \propto r^3$, the term is proportional to r^2, proportional to $A^{2/3}$.

	$-C_3 \frac{Z(Z-1)}{A^{1/3}}$:
8(b)(i)	Only terms involving Z may be different. $C_3 \frac{Z(Z-1)}{A^{1/3}}$ is different since both have different Z numbers. $\mathrm{C}_4 \frac{(A-2Z)^2}{A}$ is the same even though it involves Z. for ${}^{15}_{8}O,\ (A-2Z)^2 = (15-16)^2 = 1$ for ${}^{15}_{7}N,\ (A-2Z)^2 = (15-14)^2 = 1$
8(b)(ii)	Since only the term $C_3 \frac{Z(Z-1)}{A^{1/3}}$ differs, for ${}^{15}_{8}O,\ C_3 \frac{Z(Z-1)}{A^{1/3}} = C_3 \frac{8(8-1)}{15^{1/3}} = C_3 \frac{56}{15^{1/3}}$ for ${}^{15}_{7}N,\ C_3 \frac{Z(Z-1)}{A^{1/3}} = C_3 \frac{7(7-1)}{15^{1/3}} = C_3 \frac{42}{15^{1/3}}$ Since the value of this term for ${}^{15}_{7}N$ is smaller, the E_B will be higher. With a higher E_B, it means that ${}^{15}_{7}N$ nucleus is more tightly bound.
8(b)(iii)	${}^{15}_{8}O \rightarrow {}^{15}_{7}N + \beta^+$ Net energy change when ${}^{15}_{8}O$ decays = $C_3 \frac{56}{15^{1/3}} - C_3 \frac{42}{15^{1/3}}$ $= 0.7100 \times \frac{14}{15^{1/3}} = 4.03047$ MeV Rest Energy of $\beta^+ = m_e c^2 = 0.511$ MeV Since the net energy change is larger than the rest energy of the positron emitted, it is able to provide sufficient energy for the production of a positron. Hence, the decay is energetically possible.