NATIONAL UNIVERSITY OF SINGAPORE

PC1144 PHYSICS IV

(Semester II: AY 2008-09)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **five** **short** questions in Part I and **three** **long** questions in Part II. It comprises **eight** printed pages.
2. Answer **ALL** questions.
3. All answers are to be written on the answer books.
4. This is a **CLOSED BOOK** examination.
5. The total mark for Part I is 40 and that for Part II is 60.
6. Some useful information are given on pages 7 – 8 of this question paper.

PC1144 – PHYSICS IV

PART I

This part of the examination paper contains **five** short-answer questions from page 2 to 3. Answer **ALL** questions. The mark for each part is indicated in the square bracket.

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1. Electrons are accelerated through a 20 V potential difference, producing a mono-energetic beam. This is directed at a double-slit apparatus. A series of electron detectors are located beyond the double slit. With slit 1 alone open, 100 electrons per second are detected at all detectors. With slit 2 alone open, 900 electrons per second are detected at all detectors. Now both slits are open.

   (a) What is the de Broglie wavelength of the electrons? [3]

   (b) How many electrons per second will be detected at the centre of the interference pattern? [3]

   (c) The first minimum in the detector count occurs at detector X. How many electrons per second will be detected at this detector? [2]

2. (a) In Compton scattering, two peaks are usually present in the spectrum. Explain how these two peaks come about and write down the relationship between the wavelengths of these two peaks. [3]

   (b) Very energetic γ-rays can reach a radiation detector by Compton scattering from the surroundings, an effect known as backscattering. Show that, when the energy of the photon is very large, ($E \gg m_e c^2$ where m_e is the mass of electron), the backscattered photon has an energy of approximately 0.25 MeV, independent of the energy of the original photon, when the scattering angle is nearly 180°. [5]

3. Consider a particle of mass m with potential energy of the form

$$U(x) = \begin{cases} -U_0 \times \left(\frac{x}{a}\right) & : \quad x < 0 \\ U_0 \times \left(\frac{x}{a}\right) & : \quad x > 0 \end{cases}$$

where U_0 and a are constants.

(a) Sketch the potential energy function. [2]

(b) Use the uncertainty relations to estimate the lowest energy the particle can have. [6]

4. (a) The muon decays to an electron via the following process.

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

State the relevant conservation laws in this decay. [2]

(b) Consider a system of a proton with a muon arranged similar to a hydrogen atom, i.e., with the electron replaced by a muon. What is the muon-proton separation of the first Bohr orbit? (Hint: you should work out the reduced mass and compare to Bohr radius given in page 7.) [6]

5. A π-meson at rest decays according to

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \ .$$

(a) What is the energy carried off by the neutrino? [5]

(b) What is the velocity of the muon? [3]

(You should treat the motion of the muon relativistically and assume that the neutrino has negligible rest mass and moves off with the speed of light.)

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PART II

This part of the examination paper contains **THREE** long-answer questions from page 4 to 6. Answer **ALL** questions. The mark for each part is indicated in the square bracket.

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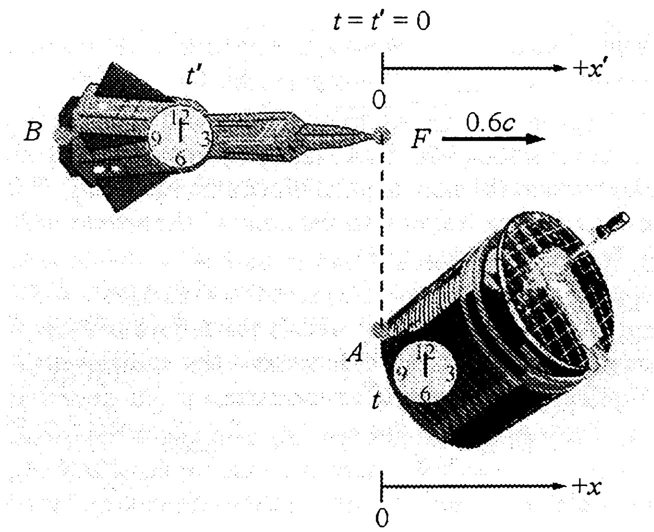
6. (a) (i) Explain clearly the meaning of time dilation and length contraction in special relativity. [4]

(ii) Starting from the Lorentz transformations between two inertial frames as given in page 7, derive the expressions for time dilation and length contraction. [6]

(b) A spaceship of length 30 m travels at $0.6c$ past a satellite. Clocks in frame S' of the spaceship and S of the satellite are synchronised within their respective frames of reference and are set to zero so that $t' = t = 0$ at the instant the front of the spaceship F passes point A on the satellite located at $x' = x = 0$. At this time a light flashes at F.



(i) What is the length of the ship as measured by an observer on the satellite? [2]

(ii) What time does the observer on the satellite read from her clock when the trailing edge of the spaceship B passes A? [2]

(iii) When the light flash reaches B at the rear of the spaceship, what is the reading t'_1 of a clock at B? [2]

(iv) What is the reading t_1 on the clock on the satellite when, according to the observer of the satellite, the flash reaches B? [4]

7. (a) An electron is confined in a one-dimensional box of length L. You may assume that the potential energy inside the box (i.e., $0 < x < L$) is zero while that outside the box is infinite.

   (i) Sketch the wavefunction and the probability density function of the third excited state ($n = 4$). [4]

   (ii) Write down the wavefunction of this state. [2]

   (iii) Deduce from the wavelength of the wavefunction that the energy of the electron in this state is given by

$$E_4 = \frac{2h^2}{mL^2} \quad .$$

[3]

(b) The potential energy cannot be really infinite outside the box. Suppose $U(x) = U_0$ outside the box where U_0 is large but finite.

   (i) Sketch the wavefunction of the third excited state of the electron in the box with this potential energy function. [3]

   (ii) By making some suitable sketches, show that the wavelength of the electron in this state must fall within $\frac{1}{2}L$ and $\frac{2}{3}L$. [3]

   (iii) Hence state the range of the energy within which the energy of the electron in this state must fall. [2]

   (iv) If $U_0 = 10h^2/(mL^2)$, predict the number of allowed bound states. [3]

8. The liquid drop model of the nucleus allows us to estimate the binding energy E_B of the many nuclei using the semi-empirical mass formula,

$$E_B = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(A-2Z)^2}{A} \pm C_5 A^{-4/3}$$

where A is the mass number, Z is the atomic number, $C_1 = 15.75$ MeV, $C_2 = 17.80$ MeV, $C_3 = 0.7100$ MeV, $C_4 = 23.66$ MeV and $C_5 = 39$ MeV.

(a) Give a physical explanation for each of the first three terms above. You should provide a physical basis for the dependence of the binding energy on A and/or Z in these terms but not the numerical values of C_1, C_2 or C_3. [8]

(b) Nuclei of the same A but different Z are known as isobars such as ${}^{15}_{8}\mathrm{O}$ and ${}^{15}_{7}\mathrm{N}$.

(i) In which of the five terms above do these two isobars differ? Explain. [3]

(ii) Explain which of these isobars should be more tightly bound. [2]

(iii) The less tightly bound isobar above may decay to the more tightly bound isobar. Predict the mode of decay and write down the equation representing this decay. Perform a calculation to show whether the decay is energetically possible. [7]

– End of Paper –

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Useful Information:

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\begin{aligned}
\text{Speed of light in vacuum, } c &= 2.998 \times 10^{8}\ \text{m/s} \\
\text{Charge of electron, } e &= 1.602 \times 10^{-19}\ \text{C} \\
\text{Planck's constant, } h &= 6.626 \times 10^{-34}\ \text{J} \cdot \text{s} \\
\text{Mass of electron, } m_e &= 9.10938 \times 10^{-31}\ \text{kg} \\
\text{Mass of proton, } m_p &= 1.67262 \times 10^{-27}\ \text{kg} \\
\text{Mass of neutron, } m_n &= 1.67493 \times 10^{-27}\ \text{kg} \\
\text{Atomic mass unit, } u &= 1.661 \times 10^{-27}\ \text{kg} \quad = \quad 931.5\ \text{MeV} \\
\text{Atomic mass of } {}^{1}_{1}\text{H} &= 1.0078250\,\text{u} \\
\text{Atomic mass of } {}^{4}_{2}\text{He} &= 4.0026033\,\text{u} \\
\text{Stefan-Boltzmann constant, } \sigma &= 5.670 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \\
\text{Wien's displacement constant, } b &= 2.898 \times 10^{-3}\ \text{m} \cdot \text{K} \\
\text{Bohr radius, } a_0 &= 5.292 \times 10^{-11}\ \text{m} \\
\text{Hydrogen ground state energy, } E_1 &= -13.61\ \text{eV} \\
\text{Lorentz Transformation: } x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}
\end{aligned}
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Table 1: Properties of some hadrons

Particle	Mass (MeV/c^2)	Charge Ratio, Q/e	Spin	Baryon Number, B	Strangeness S
Mesons					
π^0	135.0	0	0	0	0
π^+	139.6	+1	0	0	0
π^-	139.6	−1	0	0	0
K^+	493.7	+1	0	0	+1
K^-	493.7	−1	0	0	−1
K^0	497.7	0	0	0	+1
η^0	547.3	0	0	0	0
Baryons					
p	938.3	+1	1/2	1	0
n	939.6	0	1/2	1	0
Λ^0	1116	0	1/2	1	−1
Σ^+	1189	+1	1/2	1	−1
Σ^0	1193	0	1/2	1	−1
Σ^-	1197	−1	1/2	1	−1
Ξ^0	1315	0	1/2	1	−2
Ξ^-	1321	−1	1/2	1	−2
Δ^{++}	1232	+2	3/2	1	0
Ω^-	1672	−1	3/2	1	−3
${\Lambda_C}^+$	2285	+1	1/2	1	0

Table 2: Properties of leptons

Particle Name	Symbol	Anti-Particle	Mass (MeV/c^2)	L_e	L_μ	L_τ
Electron	e^-	e^+	0.511	+1	0	0
Electron neutrino	ν_e	$\bar{\nu}_e$	$< 3 \times 10^{-6}$	+1	0	0
Muon	μ^-	μ^+	105.7	0	+1	0
Muon neutrino	ν_μ	$\bar{\nu}_\mu$	< 0.19	0	+1	0
Tau	τ^-	τ^+	1777	0	0	+1
Tau neutrino	ν_τ	$\bar{\nu}_\tau$	< 18.2	0	0	+1