PC2130 Quantum Mechanics I – AY2010/2011 Semester 1 Solutions

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1)

a) $H = E\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$, $S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$, $S_z = \hbar\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$

b) $S_x = +\hbar,\ 0, -\hbar$

$$|S_x = +\hbar\rangle = |\phi_+\rangle = \frac{1}{2}\begin{pmatrix}1\\\sqrt{2}\\1\end{pmatrix}$$

$$|S_x = 0\rangle = |\phi_0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

$$|S_x = -\hbar\rangle = |\phi_-\rangle = \frac{1}{2}\begin{pmatrix}1\\-\sqrt{2}\\1\end{pmatrix}$$

c) $S_z = +\hbar,\ 0,\ -\hbar$

$$|S_z = +\hbar\rangle = |1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad |S_z = 0\rangle = |2\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}, \qquad |S_z = -\hbar\rangle = |3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

d) $S_y = +\hbar,\ 0, -\hbar$

e) $|\psi(0)\rangle = |\phi_+\rangle = \frac{1}{2}\begin{pmatrix}1\\\sqrt{2}\\1\end{pmatrix}$

f) $\Big|\psi(t)\rangle = e^{-\frac{iHt}{\hbar}}\Big|\psi(0)\rangle$

$$= \frac{1}{2}e^{-\frac{iHt}{\hbar}}\left(|1\rangle + \sqrt{2}|2\rangle + |3\rangle\right)$$

$$= \frac{1}{2}\left(|1\rangle e^{-\frac{iEt}{\hbar}} + \sqrt{2}|2\rangle + |3\rangle e^{\frac{iEt}{\hbar}}\right)$$

$$= \frac{1}{2}\begin{pmatrix}e^{-\frac{iEt}{\hbar}}\\\sqrt{2}\\e^{\frac{iEt}{\hbar}}\end{pmatrix}$$

g) $\langle\psi(t)|S_x|\psi(t)\rangle = \frac{1}{4}\cdot\frac{\hbar}{\sqrt{2}}\left(e^{\frac{iEt}{\hbar}}\sqrt{2}e^{-\frac{iEt}{\hbar}}\right)\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}e^{-\frac{iEt}{\hbar}}\\\sqrt{2}\\e^{\frac{iEt}{\hbar}}\end{pmatrix}$

$$= \hbar\cos\left(\frac{Et}{\hbar}\right)$$

$$= \langle\psi(0)|S_x|\psi(0)\rangle$$

$$= +\hbar,$$

$$\text{whenever} \frac{Et}{\hbar} = 2n\pi, \qquad \text{i.e. whenever } t = \frac{2n\pi\hbar}{E} \quad (n = 1,2,\dots)$$

h) $\langle\psi(t)|S_z|\psi(t)\rangle = \langle\psi(0)|S_z|\psi(0)\rangle = 0$ at any t.

$$Since \ \ [H, S_z] = 0 \quad \Rightarrow \quad \frac{d\langle S_z\rangle}{dt} = 0$$

that is, $\langle S_z\rangle$ is a constant of motion and does not change with t.

2)

3)

4) First, formulate the problem.

Let $t = 0$ be the time at which particles just pass the 1st SG$_Z$ device, that is, $|\psi(t = 0)\rangle = |+Z\rangle$

After flying in $B_0\hat{x}$ field for $t = \frac{l_0}{v_0}$, the state right before entering the second SG$_Z$ device is $|\psi(t)\rangle = e^{-\frac{iHt}{\hbar}}|\psi(0)\rangle$

Find out t, such that $|\langle -Z|\psi(t)\rangle|^2 = \frac{1}{4}$

Hamiltonian,

$$H = -\vec{\mu}\cdot\vec{B} = -\frac{g(-e)}{2mc}S_X = \omega_0 S_X = \frac{\omega_0\hbar}{2}(|+X\rangle\langle +X| - |-X\rangle\langle -X|)$$

$$U(t) = e^{-\frac{iHt}{\hbar}} = e^{-\frac{i\omega_0 t}{2}}|+X\rangle\langle +X| + e^{\frac{i\omega_0 t}{2}}|-X\rangle\langle -X|$$

$$\langle -Z|\psi(t)\rangle = \langle -Z|U(t)|+Z\rangle$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix} e^{-\frac{i\omega_0 t}{2}} & 0 \\ 0 & e^{\frac{i\omega_0 t}{2}}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \frac{1}{2}\left(e^{-\frac{i\omega_0 t}{2}} + e^{\frac{i\omega_0 t}{2}}\right)$$

$$= \cos\left(\frac{\omega_0 t}{2}\right) \quad [\text{in Z basis}]$$

$$|\langle -Z|\psi(t)\rangle|^2 = \frac{1}{4} = \cos^2\left(\frac{\omega_0 t}{2}\right) \Rightarrow \cos\left(\frac{\omega_0 t}{2}\right) = \frac{1}{2}$$

$$\frac{\omega_0 t}{2} = \frac{\pi}{3}, \qquad t = \frac{2\pi}{3\omega_0} \quad \Rightarrow \quad l_0 = \frac{2\pi}{3\omega_0}v_0$$

5)