NATIONAL UNIVERSITY OF SINGAPORE

PC2131 ELECTRICITY & MAGNETISM I

(Semester I: AY 2008 – 09, 29 November)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** questions and comprises **FOUR** printed pages.
2. Answer **ALL** questions.
3. Answers to the questions are to be written in the answer books.
4. A list of fomulae will be supplied.
5. This is a CLOSED BOOK examination.

Question 1:

(a) Write down the 4 Maxwell's differential equations of electromagnetism. State clearly the quantities in the equations. [4]

(b) Show that

$$\nabla\cdot(\mathbf{F}\times\mathbf{G})=(\nabla\times\mathbf{F})\cdot\mathbf{G}-\mathbf{F}\cdot(\nabla\times\mathbf{G})$$

[3]

(c) Using the above identity, or otherwise, derive Poynting's theorem:

$$\frac{\partial u}{\partial t}+\nabla\cdot\mathbf{S}=-\mathbf{E}\cdot\mathbf{J}$$

[3]

Express u and $\mathbf{S}$ in terms of $\mathbf{E}$ and $\mathbf{B}$. [2]

Explain, **briefly**, how the above equation expresses the local conservation of energy. [3]

Question 2:

(a) Derive the electric field $\mathbf{E}$ at a point $\mathbf{x}$ due to an electric dipole P of moment $\mathbf{p}$, at the origin. [4]

[*Hint:* Dipole potential, $V(\mathbf{x})=\dfrac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}$]

(b) Consider two electric dipoles P_1 and P_2. P_1 has moment $\mathbf{p}_1$ and is located at $\mathbf{x}$, while P_2 has moment $\mathbf{p}_2$ and is at the origin. Calculate the force on P_1 due to P_2. [5]

(c) Figure 1 shows an electric dipole of moment $\mathbf{p}$ at distance z_0 from a grounded conducting plane, taken to be the xy plane.

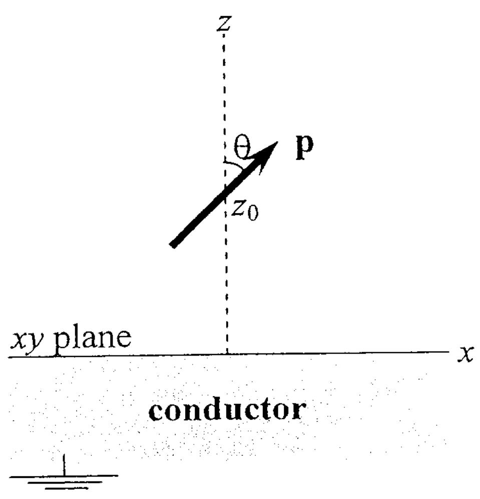

Figure 1

The direction of $\mathbf{p}$ is at an angle θ to the normal of the plane. Find the torque on $\mathbf{p}$. [6]

Question 3:

All of magnetostatics can be derived from Biot-Savart's law and the principle of superposition. These give the fundamental equation of magnetostatics:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} d^3x'$$

(a) Show that

$$\mathbf{F}\times\nabla g = (\nabla\times\mathbf{F})g - \nabla\times(\mathbf{F}g)$$

[3]

(b) Using the above identity, or otherwise, show that $\mathbf{B}(\mathbf{x})$ is solenoidal. [3]

(c) Show that

$$\nabla\cdot(g\mathbf{F}) = g\nabla\cdot\mathbf{F} + \mathbf{F}\cdot\nabla g$$

[3]

(d) Using the above identity, or otherwise, show that

$$\nabla\times\mathbf{B}(\mathbf{x}) = \mu_0\mathbf{J}(\mathbf{x})$$

[6]

[*Hint:* The Green's function of $-\nabla^2$ is given by $G(\mathbf{x}-\mathbf{x}') = \dfrac{1}{4\pi}\dfrac{1}{|\mathbf{x}-\mathbf{x}'|}$]

Question 4:

(a) Show that $\int_V (\nabla \times \mathbf{F}) d^3x = \oint_S (\hat{\mathbf{n}} \times \mathbf{F}) dA$. [5]

(b) Using the above identity, or otherwise, show that for the calculation of the vector potential

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{M}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

a material of magnetization $\mathbf{M}(\mathbf{x})$ can be replaced by a bound volume current density $\mathbf{J}_b(\mathbf{x})$ and by a bound surface current density $\mathbf{K}_b(\mathbf{x})$. Express $\mathbf{J}_b(\mathbf{x})$ and $\mathbf{K}_b(\mathbf{x})$ in terms of $\mathbf{M}(\mathbf{x})$. [5]

(c) Consider a cylinder of radius a and height h that is uniformly magnetized with magnetization $\mathbf{M} = M_0 \hat{\mathbf{k}}$.

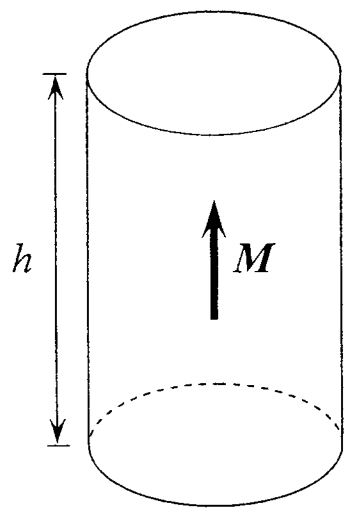

Figure 2

Calculate the magnetic field $\mathbf{B}$ on the axis of symmetry. [5]

[*Hint:* The $\mathbf{B}$ field at points on the symmetry axis of a circular loop of radius a that carries a current I is given by

$$\mathbf{B}(z) = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \hat{\mathbf{k}}$$

You may need

$$\int \frac{dx}{(1 + x^2)^{3/2}} = \frac{x}{\sqrt{1 + x^2}}$$

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