NATIONAL UNIVERSITY OF SINGAPORE

PC2131 ELECTRICITY & MAGNETISM I

(Semester II: AY 2008 – 09, 4 May)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** questions and comprises **THREE** printed pages.
2. Answer **ALL** questions.
3. Answers to the questions are to be written in the answer books.
4. A list of formulae will be supplied.
5. This is a CLOSED BOOK examination.

Question 1:

(a) Show that the curl of the gradient of a scalar function $f(\mathbf{x})$ is identically zero. [2]

(b) Show that the divergence of the curl of a vector function $\mathbf{F}(\mathbf{x})$ is identically zero. [2]

(c) Write down the 4 Maxwell's differential equations of electromagnetism. State clearly the physical quantities in the equations. [3]

(d) Show how the fields in Maxwell's equations may be represented in terms of a scalar potential $V(\mathbf{x},t)$ and a vector potential $\mathbf{A}(\mathbf{x},t)$. [2]

(e) Derive the differential equations for $V(\mathbf{x},t)$ and $\mathbf{A}(\mathbf{x},t)$, and show how they can be written in the more symmetrical form

$$\left(-\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}+\nabla^2\right)V+\frac{\partial}{\partial t}L_1=-\frac{1}{\varepsilon_0}\rho,$$

$$\left(-\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}+\nabla^2\right)\mathbf{A}-\nabla L_2=-\mu_0\mathbf{J},$$

where you are to determine L_1 and L_2. [6]

Question 2:

(a) Write down the fundamental equation of magnetostatics, which mathematically expresses the two basic principles of magnetostatics. State clearly the physical quantities in the equation. [3]

(b) From the fundamental equation of magnetostatics, show that the magnetic field is solenoidal. [3]

(c) From the fundamental equation of magnetostatics, derive the differential form of Ampere's law. [4]

Hint: The Green's function of $-\nabla^2$ is given by

$$G(\mathbf{x}-\mathbf{x}')=\frac{1}{4\pi}\frac{1}{|\mathbf{x}-\mathbf{x}'|}$$

(d) Consider a continuous distribution of currents described by the volume current density $\mathbf{J}(\mathbf{x}')$. Given that $\mathbf{x}'$'s are close to the origin, show that the vector potential due to this current distribution at a field point $\mathbf{x}$, a distance r, far away from the origin is approximately given by

$$\mathbf{A}(\mathbf{x})=\frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2},$$

where $r\hat{\mathbf{r}}=\mathbf{x}$ and

$$\mathbf{m}\equiv\frac{1}{2}\int\mathbf{x}'\times\mathbf{J}(\mathbf{x}')d^3x'.$$

[5]

Hint: You may assume the magnetic monopole term to be zero, and may find the following identities useful

$$\int x_i'J_j(\mathbf{x}')d^3x'=\frac{1}{2}\int\left[x_i'J_j(\mathbf{x}')-x_j'J_i(\mathbf{x}')\right]d^3x',$$

$$\varepsilon_{kij}\varepsilon_{klm}=\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}$$

Question 3:

(a) Show that the potential in a volume V is uniquely specified (up to an additive constant) by the solution to Poisson's equation, if either the potential or its normal derivative is specified on each surface of the volume. [7]

(b) Consider a **point charge** $+q$ located at the point (a,b,c) with two grounded, infinite conducting planes set at right angles to one another, such that it is a distance a from one and b from the other, as shown in Figure 1.

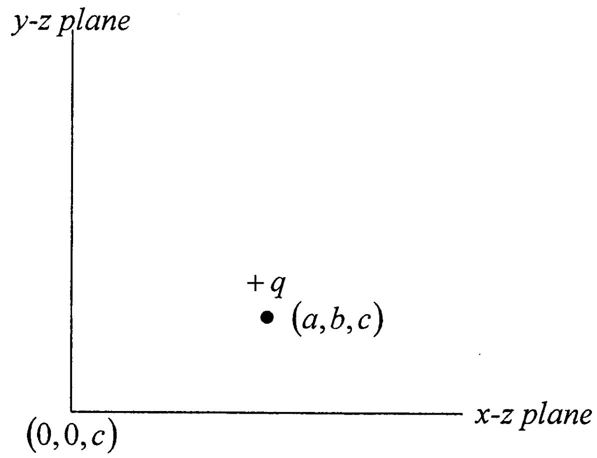

Figure 1

Calculate the charge induced on each plate. [8]

Hint: You may find the following definite integrals useful

$$\int_{-\infty}^{\infty} \frac{d\xi}{\left(\xi^2+c^2\right)^{3/2}} = \frac{2}{c^2},$$

$$\int_0^{\infty} \frac{c_1 d\xi}{c_1^2+\left(\xi \pm c_2\right)^2} = \frac{\pi}{2} \mp \arctan\frac{c_2}{c_1}$$

Question 4:

(a) Show that

$$V(r,\theta) = A + \frac{B}{r} + Cr\cos\theta + \frac{D\cos\theta}{r^2}$$

satisfies $\nabla^2 V = 0$. [5]

(b) An amount of charge Q has been deposited on a conducting sphere of radius R, and the sphere has been placed in a uniform electric field $\mathbf{E}_0$ in the positive z direction. What is the potential $V(r,\theta)$ in the region surrounding the sphere? [6]

(c) Find the electrostatic potential $V(r,\theta)$ inside a spherical volume of radius R, given that the volume is empty and that the potential at the surface $r = R$ is

$$V(R,\theta) = \frac{V_0}{2}\left(1 + 2\cos\theta + 3\cos^2\theta\right).$$

[4]

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