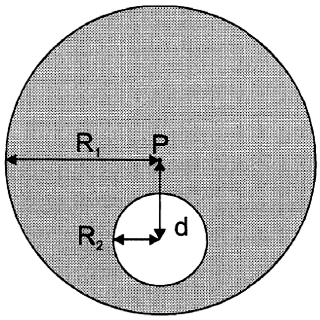
Question 1



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

The diagram above is formed by taking an infinite wire with radius R_1 and current I_1, subtracting an infinite wire of radius R_2 and current I_2,which has its centre d away from the point P. So the magnetic field at the centre is

$$B_P = B_{R_1,P} - B_{R_2,P} = 0 - \frac{\mu_0 I_2}{2\pi d} = -\frac{\mu_0 I_2}{2\pi d}$$

Where I_2 is the supposed current which flows through a wire of radius R_2. We assume the current is uniform, we can relate I_2 with I:

$$I_1 - I_2 = I$$

$$I_2 = I_1 \left(\frac{R_2^2}{R_1^2}\right)$$

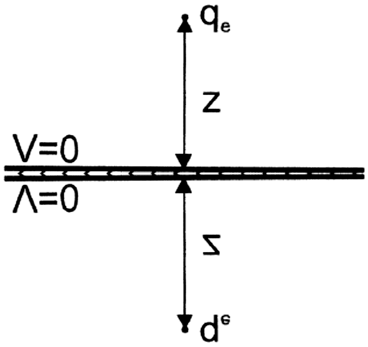
So

$$I_2 \left(\frac{R_1^2}{R_2^2}\right) - I_2 = I, \qquad I_2 = \left(\frac{R_1^2 - R_2^2}{R_2^2}\right) I$$

$\therefore \vec{B}_P = -\frac{\mu_0 I_2}{2\pi d} = -\frac{\mu_0 I}{2\pi d}\left(\frac{R_1^2 - R_2^2}{R_2^2}\right)$, rightwards if the current I is out of the page.

Question 2

Using the method of images,

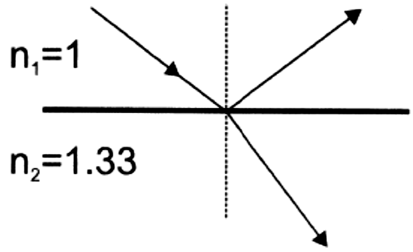


$$V = 0, \qquad y = 0$$
$$V \to 0, \qquad x^2 + y^2 \gg z^2$$

$$V = -\frac{q_e}{4\pi\epsilon_0(2z)} = -\frac{q_e}{8\pi\epsilon_0 z}$$

$$\therefore F = -q_e\frac{dV}{dz} = -\frac{q_e^2}{8\pi\epsilon_0 z^2},$$ the electron moves away from the metallic surface.

Question 3



The Brewster's angle,

$$\tan\theta_B = \frac{1.33}{1}, \qquad \theta_B = 53.06°$$

$$\theta_i = \theta_r = 53.06°$$
$$n_i \sin\theta_i = n_t \sin\theta_t, \quad \Rightarrow \quad \theta_t = 36.98°$$

$$\alpha = \frac{\cos\theta_t}{\cos\theta_i} = \frac{0.80}{0.60} = 1.33, \qquad \beta = \frac{n_2}{n_1} = 1.33$$

Using Fresnel's coefficients,

$$r^{\parallel} = \frac{\alpha-\beta}{\alpha+\beta} = 0, \qquad r^{\perp} = \frac{1-\alpha\beta}{1+\alpha\beta} = -0.28, \qquad t^{\parallel} = \frac{2}{\alpha+\beta} = 0.75, \qquad t^{\perp} = \frac{2}{1+\alpha\beta} = 0.72$$

We assume the randomly polarized light to be tilted at an angle ϕ from the vertical axis. So we have the intensities,

$$I_0 = I_p\cos\phi\,\hat{x} + I_s\sin\phi\,\hat{y}$$

$$I_t = |t^{\parallel}|^2 I_p\cos\phi\,\hat{x} + |t^{\perp}|^2 I_s\sin\phi\,\hat{y} = 0.57I_p\cos\phi\,\hat{x} + 0.52I_s\sin\phi\,\hat{y}$$
$$I_r = |r^{\parallel}|^2 I_p\cos\phi\,\hat{x} + |r^{\perp}|^2 I_s\sin\phi\,\hat{y} = 0.08I_s\sin\phi\,\hat{y}$$

Question 4

* We can consider the model of the Hall Probe. You can look up the following websites for more information:
http://sensors-actuators-info.blogspot.de/2009/08/hall-effect-sensor.html
http://www.youtube.com/watch?v=fmZJqhzVXc4

Solutions provided by: Prof Bjorn Hessmo (Q1, Q4) and John Soo (Q2, Q3)