NATIONAL UNIVERSITY OF SINGAPORE

PC2132: Classical Mechanics

(Semester I: AY 2015/16)

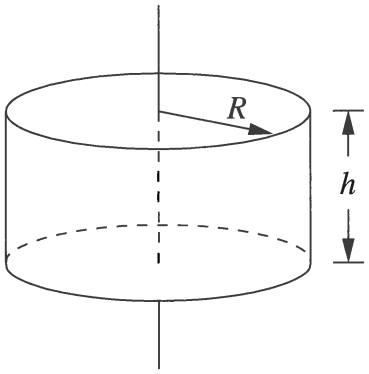
Time allowed: 2 hours

INSTRUCTIONS TO STUDENTS

1. Please write your student number only on the answer book. Do not write your name.
2. This exam paper contains **4** problems and comprises **3** printed pages.
3. You have to answer **ALL** questions.
4. Write all answers in the answer book.
5. You should begin the answers for each problem on a new page.
6. This is a CLOSED BOOK exam.
7. One Cheat Sheet (A4 size, both sides) is allowed for this exam.
8. An electronic calculator without a network connection is allowed for this exam.

Problem 1: Massive cylinder

Consider a massive cylinder with homogenous density ρ, radius R and height h as shown below:

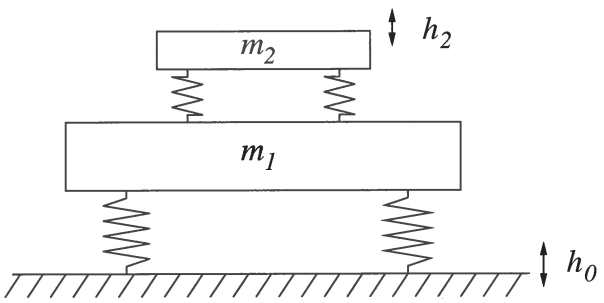


(a) Identify three rotation axes for which the inertia tensor $\boldsymbol{I}$ is diagonal. Give a reason for your answer.

(b) Calculate all non-vanishing entries of inertia tensor $\boldsymbol{I}$.

Problem 2: Vibration isolation

To protect sensitive experiments from environmental vibrations, a set of tables of masses m_1 and m_2 are stacked on each other:



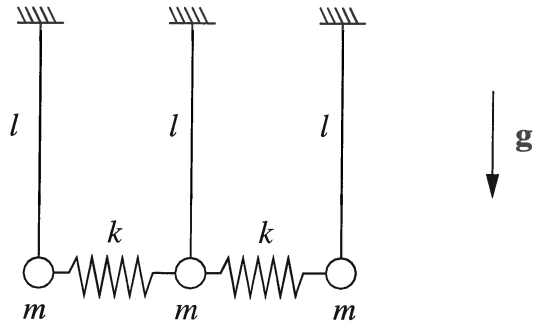
Both tables are suspended via legs that lead to a behavior of a damped harmonic oscillator, with an oscillation frequency of ω_1 and $\omega_2 = 1.5\omega_1$, respectively. The oscillation is damped with a quality factor $Q_i = \omega_i/2\beta_i = 2$ for both tables.

(a) Assume the floor vibration is harmonic with a frequency $\omega = 10\omega_1$ and an amplitude of h_0. Calculate the approximate amplitude h_2 of the vibration of table 2? Make reasonable approximations for your answer.

(b) Someone drops a sticky screwdriver with mass m_3 on table 1 out of a height h above the table surface (gravitational acceleration is g). What is the resulting change of the table height $h_1(t)$ as a function of time, assuming the table was at rest with $h_1 = 0$ before the impact? Ignore the presence of the second table for this question.

Problem 3: Three coupled pendula

Consider three pendula, each made up by a mass m and suspended from a massless string of length l, with the usual gravitational acceleration $\mathbf{g}$. The masses are connected with springs that are relaxed in the equilibrium position, and have Hook constant k:



(a) Identify a convenient set of coordinates to describe the problem, and calculate an approximate Lagrange function for the system that adequately describes small deviations from the equilibrium position.

(b) Write down the coupled equations of motion in these coordinates.

(c) Identify two modes of oscillation from the symmetry of the problem, and give the eigenvectors $\mathbf{a}_1$, $\mathbf{a}_2$ that describe the motion of all three masses for the two modes.

(d) For those two modes, calculate the oscillation (angular) frequencies ω_1 and ω_2. Give a physical reason for your answer.

(e) Construct the eigenvector $\mathbf{a}_3$ for the third mode from the first two.

(f) Calculate the corresponding angular frequency ω_3.

Problem 4: New Horizons spacecraft

A spacecraft passed by Pluto (radius $R = 1\,200\,\mathrm{km}$) with a minimal distance $d = 12\,500\,\mathrm{km}$ above its surface earlier this year.

(a) The gravitational acceleration on the surface of Pluto is about $g_p = 0.6\,\mathrm{ms}^{-2}$. Calculate the acceleration on the spacecraft in the pericenter of the trajectory.

(b) The velocity of the spacecraft (assumed to be measured in the Pluto reference frame) after the flyby is $v = 14.5\,\mathrm{km\,s}^{-1}$. What is the deflection angle ϕ of the spacecraft due to the flyby?

— End of paper — C.K.