NATIONAL UNIVERSITY OF SINGAPORE

PC2132: Classical Mechanics

(Semester I: AY 2016/17)

Time allowed: 2 hours

INSTRUCTIONS TO STUDENTS

1. Please write your student number only on the answer book. Do not write your name.
2. This exam paper contains **4** problems and comprises **3** printed pages.
3. You have to answer **ALL** questions.
4. Write all answers in the answer book.
5. You should begin the answers for each problem on a new page.
6. This is a CLOSED BOOK exam.
7. One Cheat Sheet (A4 size, both sides) is allowed for this exam.
8. An electronic calculator without a network connection is allowed for this exam.

Problem 1: Ion trap

An ion trap is a mechanism to provide a trapping potential for charged particles of the following form (assuming motion in one dimension only):

$$U_t(x) = ax^2 \quad \text{with} \quad a > 0\,.$$

(a) If the trap holds a single ion of mass m, what is its equilibrium position, and what is its frequency for small oscillations?

(b) Will there be different frequencies for large oscillations of a single ion in this potential?

(c) Now, a second ion of the same mass m is added. The two ions repel each other with a Coulomb interaction potential $U_i(x_1 - x_2) = k/|x_1 - x_2|$. Find a set of new equilibrium positions $x_{1,0}$ and $x_{2,0}$ for the two ions.

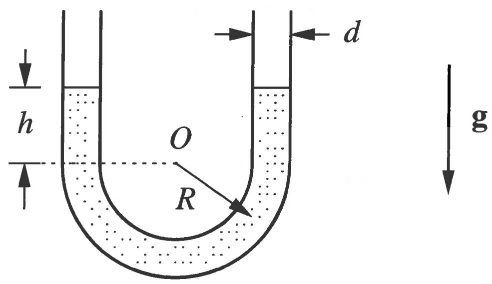
(d) Write down the coupled equations of motion for the two ions for small deviations from their equilibrium positions.

(e) What are the eigenfrequencies of this two-ion crystal in this trap?

(f) Write down a general solution for both ion positions for each of the eigenmodes.

Problem 2: U-tube

A U-shaped pipe of circular cross section with the indicated geometry is filled with an incompressible liquid of mass density ρ, and subject to gravitational acceleration **g**:



(a) Identify a suitable coordinate to describe the motion of the liquid, assuming a homogenous velocity across the tube cross section.

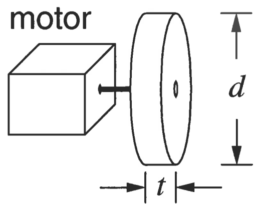
(b) Find the Lagrange function of the problem, neglecting any friction in the motion of the liquid, and assuming $d \ll R$.

(c) Find the oscillation frequency of the liquid in the tube (for sufficiently small oscillation amplitudes).

(d) Now, the tube is shaken horizontally with a given (angular) frequency ω according to $x(t) = x_0 \cos(\omega t)$, where $x(t)$ is the instantaneous horizontal position O of the U-tube center. Write down an equation of motion for the coordinate in part (a) (*No need to solve*).

Problem 3: Satellite orientation

A satellite with an onboard telescope needs to orient itself with respect to the stars. This is done with so-called reaction wheels, which are simply spinning wheels on electrical motors attached to the satellite body.



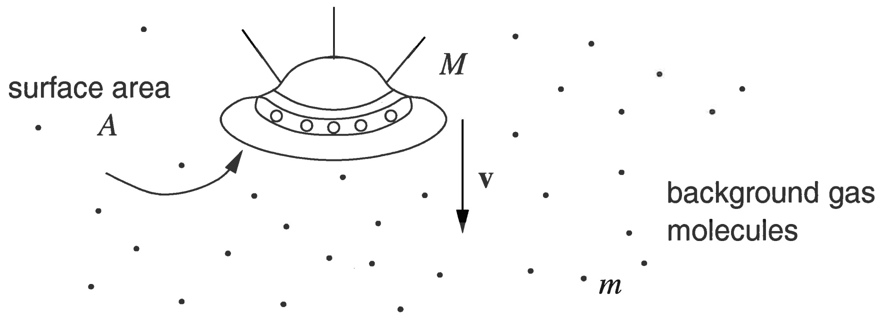
(a) Describe briefly the physical mechanism behind this orientation technique, how many reaction wheels are necessary to achieve an arbitrary orientation of the satellite. Suggest how to orient their axes.

(b) Assume a homogenous density $\rho = 1000\,\mathrm{kg/m^3}$ of a cubic satellite of length $a = 10\,\mathrm{cm}$, and calculate its inertial tensor $\underline{\underline{\mathbf{I}}}$ with respect to its center.

(c) To accomplish a telescope orientation accuracy of $\delta\phi_s = 5\,\mu\mathrm{rad}$, a motor with an accuracy of $\delta\phi_m = 1°$ is available. Calculate the necessary thickness t of a solid cylindrical reaction wheel ($\rho_w = 7000\,\mathrm{kg/m^3}$, $d = 1\,\mathrm{cm}$; neglect the inertial moment of the motor).

Problem 4: Drag in near-earth space

A spacecraft entering the atmosphere experiences a velocity-dependent force. Assume that the spacecraft moves with its (flat) base oriented normal to the flight direction, $\mathbf{v} \perp A$, as shown:



Derive a law for the friction force $\mathbf{F}(\mathbf{v})$, where $\mathbf{v}$ is velocity of the vehicle's motion with respect to the gas molecules. Assume that (i) the vehicle moves much faster than the gas molecules, (ii) its mass is much larger than the molecule of mass m, and (iii) that the gas molecules don't interact with each other in the process. The gas should have a given mass density ρ near the location of the vehicle (*Hint: consider first the interaction of a single molecule with the spacecraft*).

— End of paper — C.K.