PC2132 CLASSICAL MECHANICS
AY2008/2009 Semester 1

Suggested Solutions

Kindly note that this solution is prepared by me, Kai Ming. So, do expect alot alot of mistakes. Btw, it's not completed yet. I'm update it asap. =) Cheers!

Question 1

$$\mathbf{F} = -\frac{2k}{r^3}\hat{r}$$

$$\frac{d\mathbf{J}}{dt} = \mathbf{F} \times \hat{r} = 0$$

$\mathbf{J} =$ constant. Angular momentum is conserved.

Question 2

$$q(t,\alpha) = q(t) + \alpha\eta$$

$$\dot{q}(t,\alpha) = \dot{q}(t) + \alpha\dot{\eta}$$

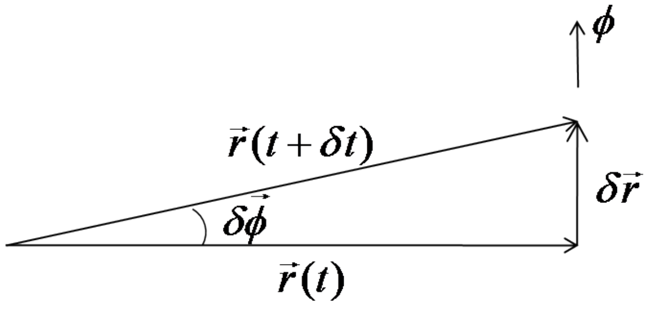
$$\left(\frac{\partial S}{\partial \alpha}\right)_{\alpha=0} = \int_{t_1}^{t_2} \sum_i \left(\frac{\partial L}{\partial q_i}\eta_i + \frac{\partial L}{\partial \dot{q}_i}\dot{\eta}_i\right) dt = 0$$

$$\begin{aligned}
\int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}_i}\dot{\eta}_i dt &= \left[\frac{\partial L}{\partial \dot{q}_i}\eta_i\right]_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\eta_i dt \\
&= 0 - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\eta_i dt
\end{aligned}$$

$$\left(\frac{\partial S}{\partial \alpha}\right)_{\alpha=0} = \int_{t_1}^{t_2} \sum_i \left(\frac{\partial L}{\partial q_i}\eta_i - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\eta_i\right) dt = 0$$

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0$$

Question 3



$$\begin{aligned}
\vec{v} &= \frac{\delta\phi}{\delta t}|\vec{r}|\hat{\phi} \\
&= |\omega| \cdot |\vec{r}|(\hat{\omega} \times \hat{r}) \\
&= \vec{\omega} \times \vec{r}
\end{aligned}$$

Question 4

$$v_0 = \sqrt{2gR}$$

Question 5

$$\begin{aligned} F &= -kx + \varepsilon e \\ &= -k\left(x - \frac{\varepsilon e}{k}\right) \end{aligned}$$

Question 6

$$\begin{aligned} \tan(\theta) &= \frac{a_{tangential}}{a_{centripetal}} \\ &= \frac{l\ddot{\theta}}{r\omega^2} \\ &= \frac{l\ddot{\theta}}{\sqrt{R^2 + l^2 - 2Rl\cos\theta}\omega^2} \end{aligned}$$

Note: $r^2 = R^2 + l^2 - 2Rl\cos\theta$

$$\ddot{\theta} = \frac{\omega^2 \tan\theta}{l}\sqrt{R^2 + l^2 - 2Rl\cos\theta}$$

Long Questions

Question 1

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a. $p_x = m\dot{x} - \dfrac{qBy}{2c}$

$p_y = m\dot{y} + \dfrac{qBx}{2c}$

$p_z = m\dot{z}$

b. $H = \dfrac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

$H = \dfrac{1}{2m}\left[\left(p_x + \dfrac{qBy}{2c}\right)^2 + \left(p_y - \dfrac{qBx}{2c}\right)^2 + p_z^2\right]$

c. $[m\dot{x}, H] = -\dfrac{qB}{2mc}$

$[m\dot{y}, H] = +\dfrac{qB}{2mc}$

$[m\dot{z}, H] = 0$

d. ???

Question 2

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a.

$$\begin{aligned} p_\varphi &= I_1\dot{\varphi}\sin^2\theta + I_3(\dot{\psi} + \dot{\varphi}\cos\theta)\cos\theta & &\rightarrow \dot{\varphi} = \frac{p_\varphi - p_\psi\cos\theta}{I_1\sin^2\theta} \\ p_\psi &= I_3(\dot{\psi} + \dot{\varphi}\cos\theta) & &\rightarrow \dot{\psi} = p_\psi\left(\frac{1}{I_3} + \frac{1}{I_1\tan^2\theta}\right) - p_\varphi\frac{1}{I_1\tan\theta\sin\theta} \\ p_\theta &= I_1\dot{\theta} & &\rightarrow \dot{\theta} = \frac{p_\theta}{I_1} \end{aligned}$$