NATIONAL UNIVERSITY OF SINGAPORE

PC2132 CLASSICAL MECHANICS

(Semester I: AY 2008-09)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **six (6) short** questions in Part I and **three (3) long** questions in Part II. It comprises **five (5)** printed pages.
2. Answer **ALL** the questions in Part I. The answers to Part I are to be written on the answer books.
3. Answer any **TWO** of the questions in Part II. The answers to Part II are to be written on the answer books.
4. This is a CLOSED BOOK examination. Students are allowed to bring in an A4-sized (both sides) sheet of notes.
5. The total mark for Part I is 48 and that for Part II is 52.

Part I: Short Answer Questions

This part contains **SIX** (6) short answer questions. Answer all questions.
The answers to the questions are to be written in the answer book(s).

1. A particle of mass m moves in a plane under the potential of $V(r) = -\frac{k}{r^2}$, $k > 0$. What physical quantities are conserved? Why?

2. The action S is defined as: $S[q] \equiv \int_{t_1}^{t_2} L(q(t), \dot{q}(t), t)dt$. Here L is the Lagrangian. The motion of a mechanical system is to make this action minimum. Derive the Lagrangian equation.

3. Use a schematic picture to show that the velocity of any point p in a rotating body in terms of the angular velocity, $\vec{\omega} \equiv d\vec{\phi}/dt$, is $\vec{v}_p = d\vec{r}/dt = \vec{\omega} \times \vec{r}$.

4. A satellite is launched from the surface of the earth with an initial velocity v_0. Find the velocity v_0 with which the satellite can escape the gravitational field of the earth.

5. A charged particle of mass m and charge $+e$ connected to a spring of constant k moves in a uniform electric field, $\vec{\varepsilon}$. Find the equilibrium point of the charged particle and the frequency of the oscillation around this point.

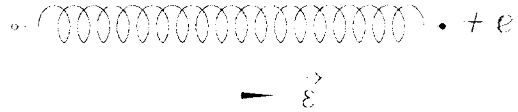

Fig 1. (Question 5 of Part I)

6. A mass m is attached to one end of a light rod of length l. The other end of the rod is pivoted so that the rod can swing in a plane. The pivot rotates in the same plane at angular velocity ω in a circle of radius R. Derive the equation of motion of the mass m.

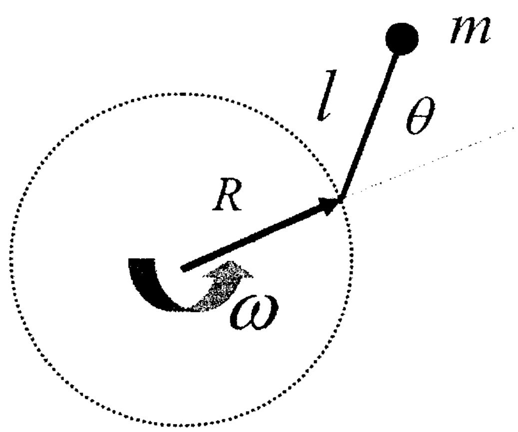

Fig. 2 (Question 6 of Part I)

--------End of Part I-------------

-Part II: Long Answer Questions -

This part contains three (3) long answer questions. Answer **ANY TWO** questions. The answers to the questions are to be written in the answer book(s)

1. The equation of motion for a particle of mass m and charge q moving in a uniform magnetic field B which points in the z-direction can be obtained from the Lagrangian

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{qB}{2c}(x\dot{y} - y\dot{x}),$$

where c is a constant.

   a. Find the momenta (p_x, p_y, p_z) conjugate to (x, y, z).
   b. Find the Hamiltonian, expressing your answer first in terms of $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ and then in terms of (x, y, z, p_x, p_y, p_z).
   c. Evaluate the Poisson brackets: $[m\dot{x}, H]$; $[m\dot{y}, H]$; $[m\dot{z}, H]$.
   d. Derive the equation of motion from the above Poisson brackets.

2. Consider the motion of a symmetric top ($I_1 = I_2$) of mass m under the gravitational force. Using the three Euler angles as generalized coordinates, the Lagrangian can be written as:

$$L = \frac{1}{2} I_1 \dot{\varphi}^2 \sin^2\theta + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_3 (\dot{\psi} + \dot{\varphi}\cos\theta)^2 - mgl\cos\theta$$

   a. Find the conjugate momenta.

   b. Find the Hamiltonian.

   c. Find the three constants of motion.

3 A double plane pendulum consists of two simple pendulums, with one pendulum suspended from the bob of the other. Both pendulums have mass m and length l, and move in the same vertical plane.

   a. Find the Lagrangian by using the generalized coordinates θ_1 and θ_2

shown in Figure 3.

b. Derive the equation of motion from the Lagrangian.

c. Simplify the equation of motion for small angle motion.
(Hints: $\sin\theta \approx \theta,\ \cos\theta \approx 1.$)

d. Find the amplitudes and frequencies of the normal oscillations.

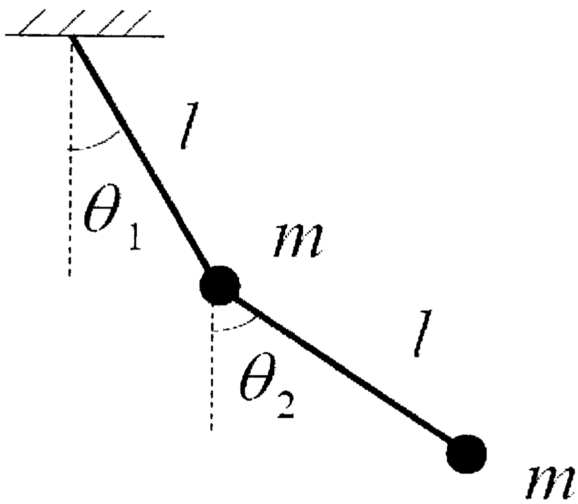

Figure 3. Double plane pendulum.

------- End of Part II ----

------ End of the Paper -----

@BWLI