NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF B. ENG. (ENGINEERING SCIENCE)

PC2230 – THERMODYNAMICS AND STATISTICAL MECHANICS

(Semester I: AY 2017-18)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name.**
2. This examination paper contains **SIX short** questions in Part I and **TWO** long questions in Part II. It comprises **SIX** printed pages.
3. Answer **ALL** the questions in this paper.
4. The answers to Part I and II are to be written on separate answer books, which are to be submitted at the end of the examination.
5. Please start each question on a new page.
6. This is a CLOSED BOOK examination.
7. The last page contains a list of formulae.
8. The total mark for Part I is 48 and that for Part II is 52.

Part I

This part of the examination paper contains six short-answer questions, which carry 8 marks each. Answer ALL questions.

1. A reversible Carnot cycle, using a perfect gas as a working substance, is performed between two heat reservoirs at temperatures T_1 and T_2. The temperature difference $T_2 - T_1 = 200$ K and the entropy change during an isothermal transformation at temperature T_1 is $\Delta S = -20$ J/K. Calculate the total work done by the Carnot Cycle.

   (8 marks)

2. A block of mass $m = 1000$ kg is initially at temperature $T_1 = 90°$C. It is then thrown into a pond, which has a temperature $T_2 = 20°$C. Calculate the total change in entropy of the universe (mass+pond). The specific heat capacity for the block is $c = 1\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

   (8 marks)

3. A system consists of N weakly interacting subsystems. Each subsystem possesses only two energy levels E_1 and E_2, which are non-degenerate with $E_2 - E_1 = 0.3$ eV. Given that the relative population of the excited state is 3×10^{-10}, calculate the total internal energy E of the system.

   (8 marks)

4. The molecules of a diatomic gas possess rotational energy levels

$$\epsilon_r = \frac{\hbar^2}{2I} r(r+1), \qquad r = 0, 1, 2, \ldots,$$

(where I is a constant), the level ϵ_r being $(2r+1)$-fold degenerate. What is the molar rotational heat capacity of nitrogen (N_2) at a temperature of 100 K, given that $I = 2.6 \times 10^{-46}$ kg m^2 for N_2?

(8 marks)

5. A large cavity with a very small hole and maintained at a temperature T is a good approximation to an ideal blackbody. Radiation can pass into or out of the cavity only through the hole. The cavity is a perfect absorber, since any radiation incident on the hole becomes trapped inside the cavity. Such a cavity at 200°C has a hole with area 4.00 mm^2. How long, in hours, does it take for the cavity to radiate 100 J of energy through the hole?

(8 marks)

6. The vapor pressure p (in millimeters of mercury) of solid ammonia is given by $\ln p = 23.03 - 3754/T$ and that of liquid ammonia by $\ln p = 19.49 - 3063/T$. Temperature is in kelvin. What is the temperature of the triple point?

(8 marks)

End of Part I

Part II

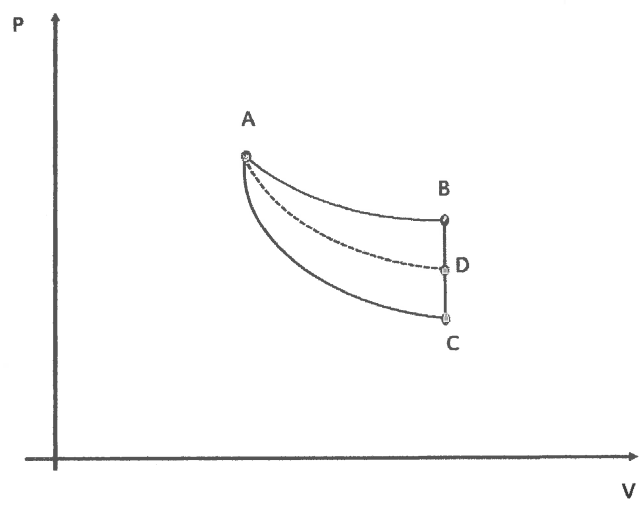
This part of the examination paper contains two (2) long-answer questions. Answer **BOTH** questions.

1. One mole of a perfect gas undergoes the following cycle: from the initial state A (T_A =560 K), there is an isothermal reversible transformation to state B. From state B to state C (T_C = 280 K), the system undergoes a reversible isovolumetric decrease in pressure. Finally, the system returns to state A through a reversible adiabatic process.

   (a) Calculate the efficiency η of the cycle.

   (6 marks)

   Consider another cyclic process, where the system undergoes an adiabatic transformation from state A to state D (T_D = 420 K). It then undergoes the same thermodynamics processes (as above) from state D to C and finally to state A. The entropy change of the universe (system + surroundings) for the AD transformation is ΔS_{uni} = 8.4 J/K.



   (b) Determine the heat capacity at constant volume C_V, and

   (8 marks)

   (c) the total work of the cycle ADCA. (6 marks)

   (d) Is the AD transformation reversible? Explain your answer.

   (6 marks)

2. Consider a system consisting of two particles, each of which can be in any one of three quantum states of respective energies ϵ, 2ϵ and 3ϵ. The system is in contact with a heat reservoir at temperature T. The particles obey Bose-Einstein statistics.

   (a) Calculate the partition function Z.

   (6 marks)

   (b) Calculate the entropy of the system.

   (6 marks)

   (c) Evaluate the average occupation numbers of the two excited states with energies 2ϵ and 3ϵ, given that $\epsilon = 10\,k_B$, $T = 20$ K and k_B is the Boltzmann constant. Give your answers rounded to 2 decimal places.

   (14 marks)

PET & LHS

Exam Formulae Sheet

Boltzmann constant $k_B = 1.38 \times 10^{-23}\ \mathrm{m^2\,kg\,s^{-2}\,K^{-1}}$

Planck constant $h = 6.626 \times 10^{-34}\ \mathrm{m^2\,kg}\,s^{-1}$

Gas constant $R = 8.31\ \mathrm{J\,mol^{-1}\,K^{-1}}$

$1\ \mathrm{atm} = 1.013 \times 10^5\ \mathrm{Pa}, \quad 1\ \mathrm{mole} = 6.022 \times 10^{23}$

$$F = E - TS, \quad G = F + PV, \quad \Omega = E - TS - \mu N$$
$$\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V + \mu \mathrm{d}N, \quad \mathrm{d}F = -S\mathrm{d}T - P\mathrm{d}V + \mu\mathrm{d}N$$
$$\mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}P + \mu\mathrm{d}N, \quad \mathrm{d}\Omega = -S\mathrm{d}T - P\mathrm{d}V - N\mathrm{d}\mu$$

For an adiabatic change of a perfect gas,

$$Tv^{\gamma-1} = \text{constant}, \quad Pv^{\gamma} = \text{constant}, \quad \text{Work done} = \frac{R\,\mathrm{d}T}{\gamma - 1}$$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\,\mathrm{d}x, \quad \Gamma(n+1) = n\Gamma(n), \quad \Gamma(1/2) = \sqrt{\pi}, \quad \Gamma(1) = 1$$

$$S = -k_B \sum_r p_r \ln p_r, \quad f(p)\,\mathrm{d}p = \frac{V 4\pi p^2}{h^3}\,\mathrm{d}p$$

Planck's blackbody radiation law: $u(\omega, T)\,\mathrm{d}\omega = \dfrac{\hbar\omega^3\,\mathrm{d}\omega}{\pi^2 c^3[\exp(\hbar\omega/k_B T) - 1]}$.

$$u(T) = aT^4, \quad a = \frac{\pi^2 k_B^4}{15\hbar^3 c^3}, \quad I = \sigma T^4, \quad \sigma = 5.67 \times 10^{-8}\ \mathrm{Jm^{-2}s^{-1}K^{-4}}$$

Stirling's formula: $\ln N! = N\ln N - N + \frac{1}{2}\ln N + \frac{1}{2}\ln(2\pi) + O\left(\frac{1}{N}\right)$

Single-particle translational partition function: $Z_1^{\mathrm{tr}} = V\left(\dfrac{2\pi m k_B T}{h^2}\right)^{3/2}$

$$E^2 = p^2c^2 + m_0^2c^4, \quad \lambda_{\mathrm{dB}} = \frac{h}{p}, \quad E = \hbar\omega = pc \ \text{(photon)}$$

Gibbs distribution: $p_{Nr} = \dfrac{\exp[\beta(\mu N - E_{Nr})]}{\mathcal{Z}}, \quad \mathcal{Z}(T,V,\mu) = \sum_{Nr}\exp[\beta(\mu N - E_{Nr})]$

$$F(T,V,N) = -k_B T \ln Z(T,V,N), \quad \Omega(T,V,\mu) = -k_B T \ln \mathcal{Z}(T,V,\mu), \quad \sum_{n=0}^{\infty}\frac{x^n}{n!} = e^x$$

$$\prod_i c_i^{\nu_i} = \prod_i [f_i(T)]^{\nu_i} \equiv K_c(T), \quad \prod_i P_i^{\nu_i} = \prod_i [k_B T f_i(T)]^{\nu_i} \equiv K_P(T), \quad f_i(T) \equiv \frac{Z_1}{V}$$

Clausius-Clapeyron Equation: $\dfrac{\mathrm{d}P}{\mathrm{d}T} = \dfrac{L}{T\Delta V}$

$$\bar{E} = -\left(\frac{\partial \ln Z}{\partial \beta}\right)_{V,N}, \quad \bar{n}_i = -\frac{1}{\beta}\left(\frac{\partial \ln Z}{\partial \varepsilon_i}\right)_{T,\varepsilon_r(r\neq i)}$$

– END OF PAPER –