NATIONAL UNIVERSITY OF SINGAPORE

PC2230 Thermodynamics and Statistical Mechanics

(Semester II: AY2009-10, May)

Time Allowed: Two Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX SHORT** questions in Part I and **THREE LONG** questions in Part II. It comprises **NINE** printed pages.
2. Answer **ALL** questions in Part I. The answers to Part I are to be written on the question paper itself and submitted at the end of the examination.
3. Answer any **TWO** of the questions in Part II. The answers to Part II are to be written on the answer books.
4. This is a CLOSED BOOK examination. Students are allowed to bring in an A4-sized (both sides) sheet of notes.
5. The total mark for Part I is 48 and that for Part II is 52.

Matriculation No.	Marks

PART I

THIS PART OF THE EXAMINATION PAPER CONTAINS SIX (6) SHORT-ANSWER QUESTIONS FROM PAGE 2 TO 7.

Answer ALL questions. The answers are to be written on this question paper itself and submitted at the end of the examination.

1. Consider a system of 6 spin $\frac{1}{2}$ dipoles in a magnetic field B.

   (i) Show that the energy of the system is determined by n, the number of dipoles which are oriented parallel to B.

   (ii) Obtain the number of microstates $\Omega(n)$ for each macrostate specified by n for $n = 0$ to 6. Comment on the results.

2. A system consists of N weakly interacting particles, each of which can be in either one of the two non-degenerate states with respective energies ε_1 and ε_2, where $\varepsilon_1 < \varepsilon_2$. Calculate the mean energy E of the system as a function of its temperature T. Determine E in the limits of very low and very high temperatures.

3. A system of N atoms, each having spin $\frac{1}{2}$ and magnetic moment μ, is located in an external magnetic field B and is in equilibrium at temperature T.

(i) Calculate the mean energy $E(T)$ and heat capacity $C(T)$ of the system. Express your results in terms of hyperbolic functions.

(ii) Find the limiting values of $E(T)$ and $C(T)$ as $T \to 0$ and $T \to \infty$.

4. Two systems A and B with respective heat capacities C_A and C_B are initially at respective temperatures T_A and T_B. The systems are now placed in thermal contact with each other and attain their final equilibrium at temperature T.

(i) Find T in terms of T_A, T_B, C_A and C_B.

(ii) Calculate the entropy changes ΔS_A of A, ΔS_B of B and ΔS of the combined system.

(iii) Show explicitly that ΔS can never be negative and that it will be zero if $T_A = T_B$. (Note $\ln x \leq x - 1$)

5. Consider a system in thermal equilibrium at temperature T.

(i) If the Hamiltonian H of the system is of the form $H = A\xi^2$, where ξ is a position or momentum coordinate, show that the mean energy of the system $\overline{A\xi^2}$ is given by $\frac{1}{2}kT$.

(ii) What is the mean energy of the system if $H = A\xi^4$?

(iii) Determine the mean energy of the system if $H = \frac{p^2}{2m} + bq^4$, where p and q are respectively the momentum and position coordinates.

6. The energy distribution of a free electron gas is given by

$$dN(\varepsilon)=\frac{4\pi V}{h^3}(2m)^{\frac{3}{2}}\varepsilon^{\frac{1}{2}}\frac{1}{e^{\beta(\varepsilon-\mu)}+1}d\varepsilon \quad .$$

(i) Show that at $T=0K$, the mean energy E and total number of electrons N of the gas are related by $E=3N\varepsilon_F/5$, where ε_F is the Fermi energy.

(ii) Find the pressure in the gas at $T=0K$. Express the result in terms of ε_F and n, the density of electrons in the gas.

(iii) Estimate the pressure exerted by the conduction electrons in a typical metal, given $n\sim 5\times10^{22}\,cm^{-3}$, $\varepsilon_F\sim 1\times10^{-11}$ erg and 1 atm $\sim 1\times10^{6}\,dyn\ cm^{-2}$. Comment on the result.

PART II

THIS PART OF THE EXAMINATION PAPER CONTAINS THREE (3) LONG QUESTIONS AND COMPRISES TWO PAGES.

Answer any TWO questions.

1. (a) Show that the pressure of a vapour in equilibrium with a liquid or solid at temperature T is approximately given by $P \approx P_o \exp(-L/RT)$. State clearly the approximations made in your derivation.

   (b) The vapour pressure P in mmHg of a solid ammonia is given by $\ln P = 23.03 - 3754/T$, while that of liquid ammonia is given by $\ln P = 19.49 - 3063/T$.

   Calculate
   (i) the temperature of the triple point of ammonia,
   (ii) the latent heats of sublimation, vaporization and melting of ammonia at the triple point.
   $\left[R = 8.31 Jmol^{-1}K^{-1}\right]$

2. The total energy density of radiation $u(T)$ within an opaque enclosure of volume V at a given temperature T is given by the Stefan-Boltzmann law $u(T) = aT^4$, where a is a constant. The radiation pressure $P(T) = u(T)/3$.

   (i) Write down the expressions for the radiation energy $E(T)$ and radiation pressure $P(T)$ in the enclosure and apply to the fundamental thermodynamic relation $TdS = dE + PdV$. Show that the entropy S is given by $S = \frac{4}{3}aT^3V$.

   (ii) Obtain expressions for the enthalpy H, the Helmholtz free energy F and Gibbs free energy G.

   (iii) Show that the chemical potential μ of the radiation is zero. Comment on this result.

3. (a) Consider a system of N particles in contact with a heat bath/particle reservoir. Show that the mean number of particles $\overline{N}$ and its standard deviation ΔN are given respectively by $\overline{N} = -\frac{\partial \Omega}{\partial \mu}$, $\Delta N = \left(-kT\frac{\partial^2 \Omega}{\partial \mu^2}\right)^{\frac{1}{2}}$, where Ω is the grand potential of the system.

(b) Write down the expression for Ω for an ideal classical gas in terms of single-particle partition function $Z_1(T,V)$. Obtain the equation of state of the gas and an explicit expression for the relative fluctuation in the number of particles in the gas. Comment on the relative fluctuation.

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