NATIONAL UNIVERSITY OF SINGAPORE

PC3231 Electricity and Magnetism 2

(Semester I: AY 2010-11)

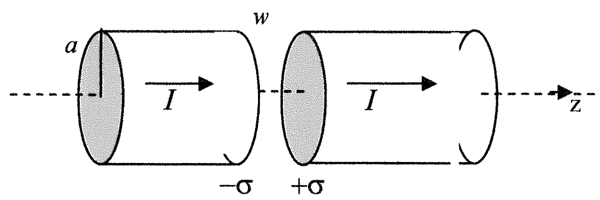
Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **4** questions and comprises **4** printed pages.
2. Answer any **3** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. One Reference Sheet (A4 size, both sides) is allowed for this examination.
6. A Table of Constants is provided.

1. <u>Parallel plate capacitor</u>

A fat wire rod of radius a carries a charging current I uniformly distributed over its cross section. A narrow gap in the wire of width $w << a$ forms a parallel-plate capacitor. The surface charge densities $\pm\sigma$ on the plates, as shown in the figure, are zero at time $t = 0$. Neglect the fringing fields in your calculation below.



(i) Determine the electric and magnetic fields in the gap as functions of the distance s from the axis z and the time t.

(ii) Determine the energy density u_{em} and the Poynting vector **S** in the gap.

(iii) Determine the total energy in the gap as a function of time.

(iv) Show that the total power flowing into the gap is equal to the rate of increase of energy in the gap.

2.

(A) <u>Gauge transformation</u>

(i) Show that in gauge transformations of Maxwell's equations in the potential formulation, we can add $\nabla\lambda$ to the vector potential $\mathbf{A}$ provided that we simultaneously subtract $\frac{\partial \lambda}{\partial t}$ from the electric potential V. Here λ is a scalar function.

(ii) Use the gauge function $\lambda = -\frac{1}{4\pi\varepsilon_o}\frac{qt}{r}$ to transform the potentials:

$$V(\mathbf{r},t) = 0$$

$$\mathbf{A}(\mathbf{r},t) = -\frac{1}{4\pi\varepsilon_o}\frac{qt}{r^2}\hat{\mathbf{r}}$$

for a stationary point charge q.

(iii) Is the set of potentials in part (ii) in Coulomb gauge or Lorentz gauge?

(B) Angular momentum

Consider an iron sphere of radius R carries a charge Q and a uniform magnetization $\mathbf{M} = M\,\hat{\mathbf{z}}$ where the magnetic fields are

$$\mathbf{B} = \begin{cases} \frac{2}{3}\mu_o M\,\hat{\mathbf{z}} & r < R \\ \frac{\mu_o}{4\pi}\frac{m}{r^3}\left[2\cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}\right] & r > R \end{cases} \qquad m = \frac{4}{3}\pi R^3 M$$

Write down the corresponding electric fields. Hence determine the total angular momentum stored in the field with reference to the center of the iron sphere.

3. Electromagnetic waves in conductors

Consider the case when an electromagnetic plane wave traveling in a non-conducting medium impinges at normal incidence on a conducting medium with conductivity σ.

(i) Given the plane wave solutions for electric field $\mathbf{E}$ in the conducting medium as

$$\mathbf{E} = \mathbf{E}_o e^{-k_- z} e^{i(k_+ z - \omega t)}$$

$$k_\pm = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} \pm 1\right]^{1/2}$$

Show that the in the conductor the magnetic field lags behind the electric field.

(ii) Derive an expression for the reflected electric field from the conducting medium.

(iii) Calculate the reflection coefficient for visible light (frequency $\omega = 5 \times 10^{15}\ s^{-1}$) at a silver surface. For silver, take $\mu \sim \mu_0$ and $\sigma = 6 \times 10^7\ (\Omega m)^{-1}$

4. Point charge in motion

The fields of a point charge q in arbitrary motion with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ are described by

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{r}{(\mathbf{r}\cdot\mathbf{u})^3}\left[(c^2 - v^2)\mathbf{u} + \mathbf{r}\times(\mathbf{u}\times\mathbf{a})\right]$$

where $\mathbf{u} = \hat{\mathbf{r}}c - \mathbf{v}$ and $\mathbf{r}$ is the vector from the point charge to the observer,

and $\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\hat{\mathbf{r}}\times\mathbf{E}(\mathbf{r},t)$

(i) Derive an expression for the Poynting vector $\mathbf{S}$ for a point charge in slow motion ($v << c$).

(ii) Suppose $\mathbf{v}$ and $\mathbf{a}$ are collinear. Sketch the angular dependence of radiated power for $v << c$. How does the angular dependence compare in appearance to the case when v is very large?

(iii) Show that for a point charge in slow motion ($v << c$), the total power radiated P is given approximately by the Larmor formula:

$$P = \frac{\mu_0 q^2 \mathrm{a}^2}{6\pi\mathrm{c}}$$

(iv) Suppose an electron decelerated at a constant rate a from some initial velocity v_0 down to zero. What fraction of its initial kinetic energy is lost to radiation? Assume $v_0 << c$ so that the Larmor formula can be applied.

~ End of Examination Paper ~

TSH